Complete Solutions Manual to Accompany

Calculus: An Applied Approach

TENTH EDITION

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Functions, Graphs, and Limits

Section 1.1 The Cartesian Plane and the Distance Formula

Skills Warm Up

1.
$$\begin{aligned}\sqrt{(3-6)^2+[1-(-5)]^2} &= \sqrt{(-3)^2+6^2}\\ &= \sqrt{9+36}\\ &= \sqrt{45}\\ &= 3\sqrt{5}\end{aligned}$$

2.
$$\begin{aligned}\sqrt{(-2-0)^2+[-7-(-3)]^2} &= \sqrt{(-2)^2+(-4)^2}\\ &= \sqrt{4+16}\\ &= \sqrt{20}\\ &= 2\sqrt{5}\end{aligned}$$

3. $\dfrac{5+(-4)}{2} = \dfrac{1}{2}$

4. $\dfrac{-3+(-1)}{2} = \dfrac{-4}{2} = -2$

5. $\sqrt{27}+\sqrt{12} = 3\sqrt{3}+2\sqrt{3} = 5\sqrt{3}$

6. $\sqrt{8}-\sqrt{18} = 2\sqrt{2}-3\sqrt{2} = -\sqrt{2}$

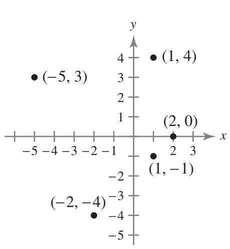
7.
$$\begin{aligned}\frac{x+(-5)}{2} &= 7\\ x+(-5) &= 14\\ x &= 19\end{aligned}$$

8.
$$\begin{aligned}\frac{-7+y}{2} &= -3\\ -7+y &= -6\\ y &= 1\end{aligned}$$

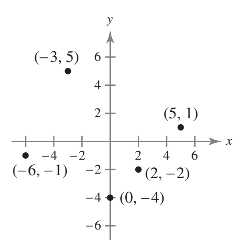
9.
$$\begin{aligned}\sqrt{(3-x)^2+(7-4)^2} &= \sqrt{45}\\ \left(\sqrt{(3-x)^2+(7-4)^2}\right)^2 &= \left(\sqrt{45}\right)^2\\ (3-x)^2+(7-4)^2 &= 45\\ (3-x)^2+3^2 &= 45\\ (3-x)^2+9 &= 45\\ (3-x)^2 &= 36\\ 3-x &= \pm 6\\ -x &= -3\pm 6\\ x &= 3\mp 6\\ x &= -3,\ 9\end{aligned}$$

10.
$$\begin{aligned}\sqrt{(6-2)^2+(-2-y)^2} &= \sqrt{52}\\ \left(\sqrt{(6-2)^2+(-2-y)^2}\right)^2 &= \left(\sqrt{52}\right)^2\\ (6-2)^2+(-2-y)^2 &= 52\\ 4^2+(-2-y)^2 &= 52\\ 16+(-2-y)^2 &= 52\\ (-2-y)^2 &= 36\\ -2-y &= \pm 6\\ -y &= \pm 6+2\\ y &= \mp 6-2\\ y &= -8,\ 4\end{aligned}$$

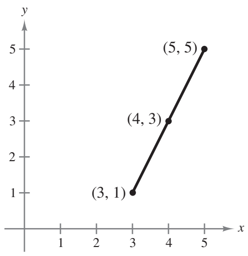
1.



2.



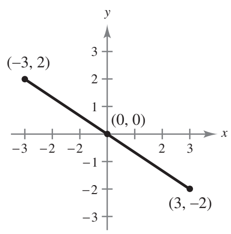
3. (a)



(b) $d = \sqrt{(5-3)^2 + (5-1)^2} = \sqrt{4+16} = 2\sqrt{5}$

(c) Midpoint $= \left(\dfrac{3+5}{2}, \dfrac{1+5}{2}\right) = (4, 3)$

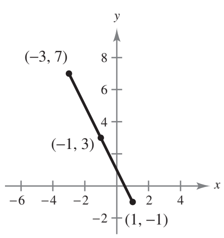
4. (a)



(b) $d = \sqrt{(-3-3)^2 + (2+2)^2} = \sqrt{36+16} = 2\sqrt{13}$

(c) Midpoint $= \left(\dfrac{-3+3}{2}, \dfrac{2+(-2)}{2}\right) = (0, 0)$

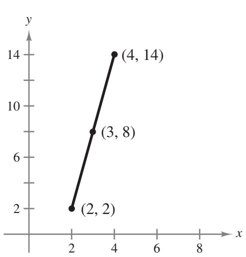
5. (a)



(b) $d = \sqrt{(-3-1)^2 + (7+1)^2} = \sqrt{16+64} = 4\sqrt{5}$

(c) Midpoint $= \left(\dfrac{-3+1}{2}, \dfrac{7-1}{2}\right) = (-1, 3)$

6. (a)



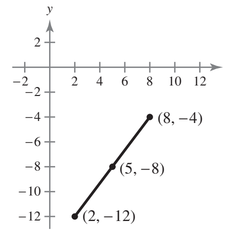
(b) $d = \sqrt{(4-2)^2 + (14-2)^2}$

$= \sqrt{4+144}$

$= 2\sqrt{37}$

(c) Midpoint $= \left(\dfrac{2+4}{2}, \dfrac{2+14}{2}\right) = (3, 8)$

7. (a)



(b) $d = \sqrt{(8-2)^2 + \left(-4-(-12)^2\right)}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36+64}$

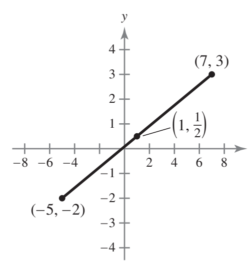
$= \sqrt{100} = 10$

(c) Midpoint $= \left(\dfrac{2+8}{2}, \dfrac{(-12)+(-4)}{2}\right)$

$= \left(\dfrac{10}{2}, \dfrac{-16}{2}\right)$

$= (5, -8)$

8. (a)



(b) $d = \sqrt{(7-(-5))^2 + \left(3-(-2)^2\right)}$

$= \sqrt{12^2 + 5^2}$

$= \sqrt{144+25}$

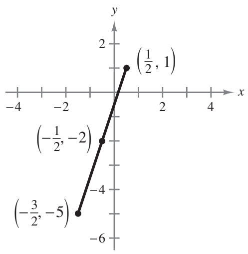
$= \sqrt{169} = 13$

(c) Midpoint $= \left(\dfrac{7+(-5)}{2}, \dfrac{3+(-2)}{2}\right)$

$= \left(\dfrac{2}{2}, \dfrac{1}{2}\right)$

$= \left(1, \dfrac{1}{2}\right)$

9. (a)



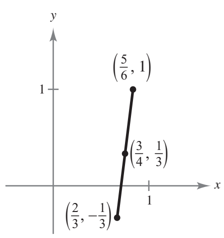
(b) $d = \sqrt{\left[(3/2) - (1/2)\right]^2 + (5 - 1)^2}$

$= \sqrt{4 + 36}$

$= 2\sqrt{10}$

(c) Midpoint $= \left(\frac{(1/2) + (-3/2)}{2}, \frac{1 + (-5)}{2}\right) = \left(-\frac{1}{2}, -2\right)$

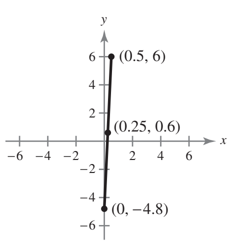
10. (a)



(b) $d = \sqrt{\left(\frac{5}{6} - \frac{2}{3}\right)^2 + \left(1 + \frac{1}{3}\right)^2} = \sqrt{\frac{1}{36} + \frac{16}{9}} = \frac{\sqrt{65}}{6}$

(c) Midpoint $= \left(\frac{(5/6) + (2/3)}{2}, \frac{1 - (1/3)}{2}\right) = \left(\frac{3}{4}, \frac{1}{3}\right)$

11. (a)



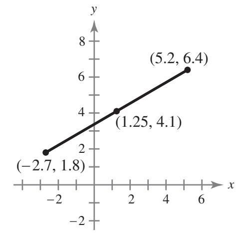
(b) $d = \sqrt{(0.5 - 0)^2 + (6 - (-4.8))^2}$

$= \sqrt{0.25 + 116.64}$

$= \sqrt{116.89}$

(c) Midpoint $= \left(\frac{0 + 0.5}{2}, \frac{-4.8 + 6}{2}\right) = (0.25, 0.6)$

12. (a)



(b) $d = \sqrt{(-2.7 - 5.2)^2 + (1.8 - 6.4)^2}$

$= \sqrt{62.41 + 21.16}$

$= \sqrt{83.57}$

(c) Midpoint $= \left(\frac{5.2 + (-2.7)}{2}, \frac{6.4 + 1.8}{2}\right)$

$= (1.25, 4.1)$

13. (a) $a = 4$

$b = 3$

$c = \sqrt{(4 - 0)^2 + (3 - 0)^2} = \sqrt{16 + 9} = 5$

(b) $a^2 + b^2 = 16 + 9 = 25 = c^2$

14. (a) $a = \sqrt{(13 - 1)^2 + (1 - 1)^2} = \sqrt{144 + 0} = 12$

$b = \sqrt{(13 - 13)^2 + (6 - 1)^2} = \sqrt{0 + 25} = 5$

$c = \sqrt{(13 - 1)^2 + (6 - 1)^2} = \sqrt{144 + 25} = 13$

(b) $a^2 + b^2 = 144 + 25 = 169 = c^2$

15. (a) $a = 10$

$b = 3$

$c = \sqrt{(7 + 3)^2 + (4 - 1)^2} = \sqrt{100 + 9} = \sqrt{109}$

(b) $a^2 + b^2 = 100 + 9 = 109 = c^2$

16. (a) $a = \sqrt{(6 - 2)^2 + (-2 + 2)^2} = \sqrt{16 + 0} = 4$

$b = \sqrt{(2 - 2)^2 + (5 + 2)^2} = \sqrt{0 + 49} = 7$

$c = \sqrt{(2 - 6)^2 + (5 + 2)^2} = \sqrt{16 + 49} = \sqrt{65}$

(b) $a^2 + b^2 = 16 + 49 = 65 = c^2$

17. $d_1 = \sqrt{(3-0)^2 + (7-1)^2}$

$= \sqrt{9 + 36}$

$= \sqrt{45}$

$= 3\sqrt{5}$

$d_2 = \sqrt{(4-0)^2 + (-1-1)^2}$

$= \sqrt{16 + 4}$

$= \sqrt{20}$

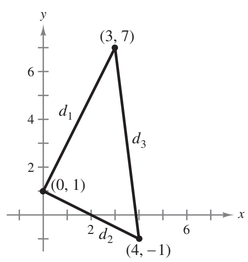
$= 2\sqrt{5}$

$d_3 = \sqrt{(3-4)^2 + [7-(-1)]^2}$

$= \sqrt{1 + 64}$

$= \sqrt{65}$

Because $d_1^2 + d_2^2 = d_3^2$, the figure is a right triangle.



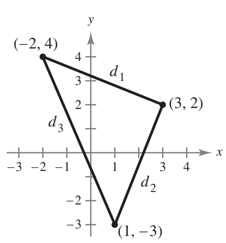
18. $a = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{25 + 4} = \sqrt{29}$

$b = \sqrt{(3-1)^2 + (2+3)^2} = \sqrt{4 + 25} = \sqrt{29}$

$c = \sqrt{(-2-1)^2 + (4+3)^2} = \sqrt{9 + 49} = \sqrt{58}$

Because $a = b$ the figure is an isosceles triangle.

[**Note:** It is also a right triangle since $a^2 + b^2 = c^2$.]



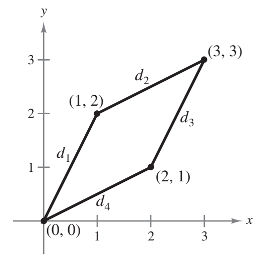
19. $d_1 = \sqrt{(1-0)^2 + (2-0)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_2 = \sqrt{(3-1)^2 + (3-2)^2} = \sqrt{4 + 1} = \sqrt{5}$

$d_3 = \sqrt{(2-3)^2 + (1-3)^2} = \sqrt{1 + 4} = \sqrt{5}$

$d_4 = \sqrt{(0-2)^2 + (0-1)^2} = \sqrt{4 + 1} = \sqrt{5}$

Because $d_1 = d_2 = d_3 = d_4$, the figure is a parallelogram.



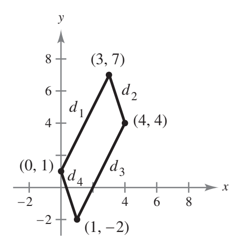
20. $a = \sqrt{(3-0)^2 + (7-1)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$b = \sqrt{(3-4)^2 + (7-4)^2} = \sqrt{1 + 9} = \sqrt{10}$

$c = \sqrt{(4-1)^2 + (4+2)^2} = \sqrt{9 + 36} = 3\sqrt{5}$

$d = \sqrt{(1-0)^2 + (-2-1)^2} = \sqrt{1 + 9} = \sqrt{10}$

Because $a = c$ and $b = d$, the figure is a parallelogram.



21. $d = \sqrt{(x-1)^2 + (-4-0)^2} = 5$

$\sqrt{x^2 - 2x + 17} = 5$

$x^2 - 2x + 17 = 25$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

22. $d = \sqrt{(x-2)^2 + (2+1)^2} = 5$

$\sqrt{x^2 - 4x + 13} = 5$

$x^2 - 4x + 13 = 25$

$x^2 - 4x - 12 = 0$

$(x+2)(x-6) = 0$

$x = -2, 6$

23. $d = \sqrt{\left(-3 - (-5)\right)^2 + (y - 0)^2} = 8$

$$\sqrt{4 + y^2} = 8$$
$$4 + y^2 = 64$$
$$y^2 = 60$$
$$y = \pm\sqrt{60}$$
$$y = \pm 2\sqrt{15}$$

24. $d = \sqrt{(4 - 4)^2 + \left(y - (-6)\right)^2} = 8$

$$\sqrt{(y + 6)^2} = 8$$
$$(y + 6)^2 = 64$$
$$y + 6 = \pm 8$$
$$y = -6 \pm 8$$
$$y = -14, 2$$

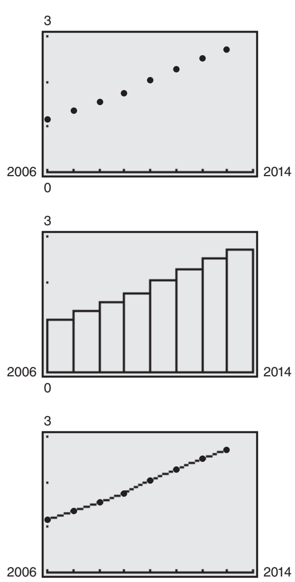
25. $d = \sqrt{(50 - 12)^2 + (42 - 18)^2}$

$$= \sqrt{38^2 + 24^2}$$
$$= \sqrt{2020}$$
$$= 2\sqrt{505} \approx 44.9 \text{ yd}$$

26. $d = \sqrt{(33 - 12)^2 + (37 - 18)^2}$

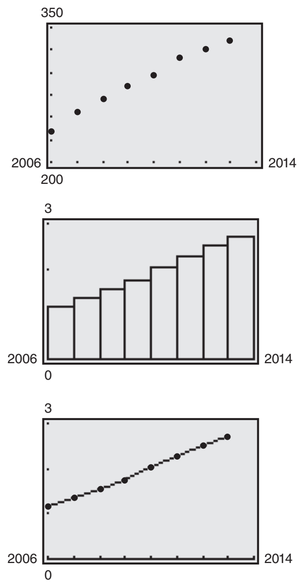
$$= \sqrt{21^2 + 19^2}$$
$$= \sqrt{441 + 361}$$
$$= \sqrt{802} \approx 28.3 \text{ yd}$$

27.



The numbers of individuals using the Internet increased each year from 2006 through 2013.

28.



The numbers of cellular telephone subscribers increased each year from 2006 through 2013.

29. (a) March 2013: 14,500

July 2013: 15,500

July 2014: 16,500

(b) December 2013: 16,600

January 2014: 15,750

Decrease: $|16,600 - 15,750| = 850$

Percent decrease: $\dfrac{850}{16,600} \approx 0.051 = 5.1\%$

30. (a) 2007: \$218,000

2009: \$172,000

2012: \$178,000

(b) 2011: \$168,000

2012: \$178,000

Increase: $178,000 - 168,000 = 10,000$

Percent increase: $\dfrac{10,000}{168,000} \approx 0.0595$

$$\approx 6.0\%$$

31. (a) Revenue $= \left(\dfrac{2011 + 2013}{2}, \dfrac{784.5 + 1266.7}{2}\right)$

$= (2012, 1025.6)$

Revenue estimate for 2012: \$1025.6 million

Profit $= \left(\dfrac{2011 + 2013}{2}, \dfrac{50.4 + 71.6}{2}\right)$

$= (2012, 61.0)$

Profit estimate for 2012: \$61.0 million

(b) Actual 2012 revenue: \$1040.5 million

Actual 2012 profit: \$57.3 million

(c) Yes, the revenue and profit increased in a linear pattern from 2011 to 2013.

(d) 2011 expense: $784.5 - 50.4 = \$734.1$ million

2012 expense: $1040.5 - 57.3 = \$983.2$ million

2013 expense: $1266.7 - 71.6 = \$1195.1$ million

(e) Answers will vary.

32. (a) Revenue $= \left(\dfrac{2011 + 2013}{2}, \dfrac{40.9 + 45.0}{2}\right)$

$= (2012, 42.95)$

Revenue estimate for 2012: \$42.95 billion

Profit $= \left(\dfrac{2011 + 2013}{2}, \dfrac{4.8 + 6.1}{2}\right)$

$= (2012, 5.45)$

Profit estimate for 2012: \$5.45 billion

(b) Actual 2012 revenue: \$42.3 billion

Actual 2012 profit: \$5.7 billion

(c) Yes, the revenue and profit increased in a linear pattern from 2011 to 2013.

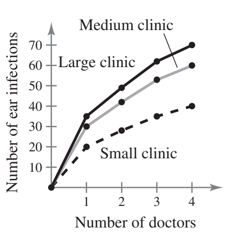
(d) 2011 expense: $40.9 - 4.8 = \$36.1$ billion

2012 expense: $42.3 - 5.7 = \$36.6$ billion

2013 expense: $45.0 - 6.1 = \$38.9$ billion

(e) Answers will vary.

33. (a)



(b) The larger the clinic, the more patients a doctor can treat.

34. (a) 500 pickups were sold in 2011.

(b) About 400 pickups were sold in 2013.

(c) The number of pickups sold each year is decreasing.

35. The vertex $(-3, -1)$ is translated to $(-6, -6)$.

The vertex $(0, 0)$ is translated to $(-3, -5)$.

The vertex $(-1, -2)$ is translated to $(-4, -7)$.

36. The vertex $(0, 2)$ is translated to $(2, 6)$.

The vertex $(1, 3)$ is translated to $(3, 7)$.

The vertex $(3, 1)$ is translated to $(5, 5)$.

The vertex $(2, 0)$ is translated to $(4, 4)$.

37. Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

The point one-fourth of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from (x_1, y_1) to $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, which is

$$\left(\frac{x_1 + \frac{x_1 + x_2}{2}}{2}, \frac{y_1 + \frac{y_1 + y_2}{2}}{2}\right) = \left(\frac{3x_1 + x_2}{4}, \frac{3y_1 + y_2}{4}\right).$$

The point three-fourths of the way between (x_1, y_1) and (x_2, y_2) is the midpoint of the line segment from $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ to (x_2, y_2), which is

$$\left(\frac{\frac{x_1 + x_2}{2} + x_2}{2}, \frac{\frac{y_1 + y_2}{2} + y_2}{2}\right) = \left(\frac{x_1 + 3x_2}{4}, \frac{y_1 + 3y_2}{4}\right).$$

Thus,

$\left(\dfrac{3x_1 + x_2}{4}, \dfrac{3y_1 + y_2}{4}\right), \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$, and $\left(\dfrac{x_1 + 3x_2}{4}, \dfrac{y_1 + 3y_2}{4}\right)$

are the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four equal parts.

38. (a) $\left(\frac{3(1)+4}{4}, \frac{3(-2)-1}{4}\right) = \left(\frac{7}{4}, -\frac{7}{4}\right)$

$\left(\frac{1+4}{2}, \frac{-2-1}{2}\right) = \left(\frac{5}{2}, -\frac{3}{2}\right)$

$\left(\frac{1+3(4)}{4}, \frac{-2+3(-1)}{4}\right) = \left(\frac{13}{4}, -\frac{5}{4}\right)$

(b) $\left(\frac{3(-2)+0}{4}, \frac{3(-3)+0}{4}\right) = \left(-\frac{3}{2}, -\frac{9}{4}\right)$

$\left(\frac{-2+0}{2}, \frac{-3+0}{2}\right) = \left(-1, -\frac{3}{2}\right)$

$\left(\frac{-2+3(0)}{4}, \frac{-3+3(0)}{4}\right) = \left(-\frac{1}{2}, -\frac{3}{4}\right)$

39. To show $\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$ is a point of trisection of the line segment joining (x_1, y_1) and (x_2, y_2), we must show that $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$.

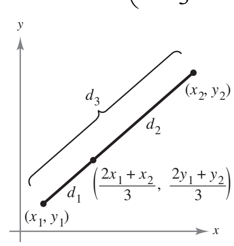
$$\begin{aligned} d_1 &= \sqrt{\left(\frac{2x_1+x_2}{3} - x_1\right)^2 + \left(\frac{2y_1+y_2}{3} - y_1\right)^2} \\ &= \sqrt{\left(\frac{x_2-x_1}{3}\right)^2 + \left(\frac{y_2-y_1}{3}\right)^2} \\ &= \frac{1}{3}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \end{aligned}$$

$$\begin{aligned} d_2 &= \sqrt{\left(x_2 - \frac{2x_1+x_2}{3}\right)^2 + \left(y_2 - \frac{2y_1+y_2}{3}\right)^2} \\ &= \sqrt{\left(\frac{2x_2-2x_1}{3}\right)^2 + \left(\frac{2y_2-2y_1}{3}\right)^2} \\ &= \frac{2}{3}\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \end{aligned}$$

$$d_3 = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Therefore, $d_1 = \frac{1}{2}d_2$ and $d_1 + d_2 = d_3$. The midpoint of the line segment joining $\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$ and (x_2, y_2) is

$$\begin{aligned} \text{Midpoint} &= \left(\frac{\frac{2x_1+x_2}{3} + x_2}{2}, \frac{\frac{2y_1+y_2}{3} + y_2}{2}\right) \\ &= \left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right). \end{aligned}$$



40. (a) $\left(\frac{2(1)+4}{3}, \frac{2(-2)+1}{3}\right) = (2, -1)$

$\left(\frac{1+2(4)}{3}, \frac{-2+2(1)}{3}\right) = (3, 0)$

(b) $\left(\frac{2(-2)+0}{3}, \frac{2(-3)+0}{3}\right) = \left(-\frac{4}{3}, -2\right)$

$\left(\frac{-2+2(0)}{3}, \frac{-3+2(0)}{3}\right) = \left(-\frac{2}{3}, -1\right)$

Section 1.2 Graphs of Equations

Skills Warm Up

1. $$\begin{aligned} 5y - 12 &= x \\ 5y &= x + 12 \\ y &= \frac{x + 12}{5} \end{aligned}$$

2. $$\begin{aligned} -y &= 15 - x \\ y &= x - 15 \end{aligned}$$

3. $$\begin{aligned} x^3y + 2y &= 1 \\ y(x^3 + 2) &= 1 \\ y &= \frac{1}{x^3 + 2} \end{aligned}$$

4. $$\begin{aligned} x^2 + x - y^2 - 6 &= 0 \\ -y^2 &= 6 - x^2 - x \\ y^2 &= x^2 + x - 6 \\ y &= \sqrt{x^2 + x - 6} \end{aligned}$$

5. $$\begin{aligned} (x - 2)^2 + (y + 1)^2 &= 9 \\ (y + 1)^2 &= 9 - (x - 2)^2 \\ y + 1 &= \sqrt{9 - (x - 2)^2} \\ y &= \left(\sqrt{9 - (x - 2)^2}\right) - 1 \\ &= \sqrt{9 - (x^2 - 4x + 4)} - 1 \\ &= \sqrt{5 + 4x - x^2} - 1 \end{aligned}$$

6. $$\begin{aligned} (x + 6)^2 + (y - 5)^2 &= 81 \\ (y - 5)^2 &= 81 - (x + 6)^2 \\ y - 5 &= \sqrt{81 - (x + 6)^2} \\ y &= 5 + \sqrt{81 - (x + 6)^2} \\ &= 5 + \sqrt{81 - (x^2 + 12x + 36)} \\ &= 5 + \sqrt{45 - 12x - x^2} \end{aligned}$$

7. $y = 5(-2) = -10$

8. $y = 3(3) - 4 = 5$

9. $$\begin{aligned} y &= 4(0.5)^2 - 7 \\ &= 4(0.25) - 7 \\ &= 1 - 7 \\ &= -6 \end{aligned}$$

10. $$\begin{aligned} y &= 9\left(\tfrac{1}{3}\right)^2 + 9\left(\tfrac{1}{3}\right) - 5 \\ &= 9\left(\tfrac{1}{9}\right) + 9\left(\tfrac{1}{3}\right) - 5 \\ &= 1 + 3 - 5 = -1 \end{aligned}$$

11. $x^2 - 3x + 2$
$(x - 1)(x - 2)$

12. $x^2 + 5x + 6$
$(x + 2)(x + 3)$

13. $y^2 - 3y + \frac{9}{4}$
$\left(y - \frac{3}{2}\right)^2$

14. $y^2 - 7y + \frac{49}{4}$
$\left(y - \frac{7}{2}\right)^2$

1. The graph of $y = x - 2$ is a straight line with y-intercept at $(0, -2)$. So, it matches (e).

2. The graph of $y = -\frac{1}{2}x + 2$ is a straight line with y-intercept at $(0, 2)$. So, it matches (b).

3. The graph of $y = x^2 + 2x$ is a parabola opening up with vertex at $(-1, -1)$. So, it matches (c).

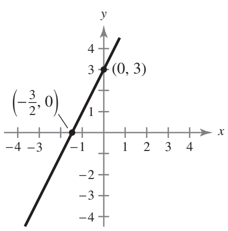
4. The graph of $y = \sqrt{9 - x^2}$ is a semicircle with intercepts $(0, 3)$, $(3, 0)$, and $(-3, 0)$. So, it matches (f).

5. The graph of $y = |x| - 2$ has a y-intercept at $(0, -2)$ and has x-intercepts at $(-2, 0)$ and $(2, 0)$.
So, it matches (a).

6. The graph of $y = x^3 - x$ has intercepts at $(0, 0)$, $(1, 0)$, and $(-1, 0)$. So, it matches (d).

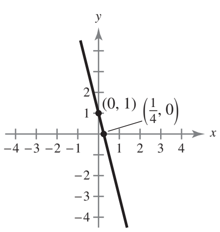
7. $y = 2x + 3$

x	-2	$-\frac{3}{2}$	-1	0	1	2
y	-1	0	1	3	5	7



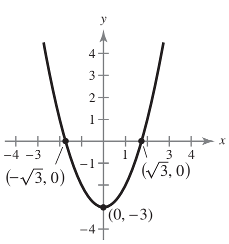
8. $y = 1 - 4x$

x	-1	0	$\frac{1}{4}$	1	2
y	5	1	0	-3	-7



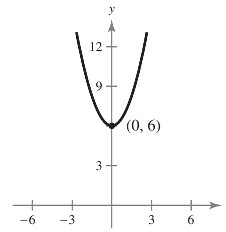
9. $y = x^2 - 3$

x	-2	-1	0	1	2	3
y	1	-2	-3	-2	1	6



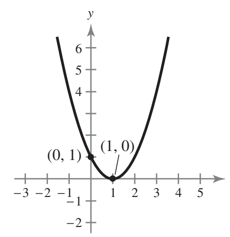
10. $y = x^2 + 6$

x	-2	-1	0	1	2
y	10	7	6	7	10



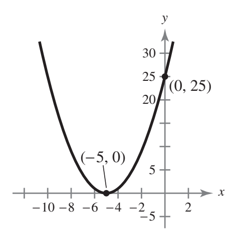
11. $y = (x - 1)^2$

x	-2	-1	0	1	2
y	9	4	1	0	1



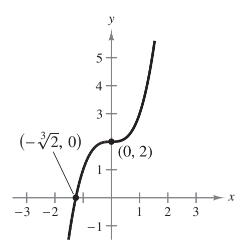
12. $y = (x + 5)^2$

x	-6	-5	-4	-3	-2
y	1	0	1	2	9



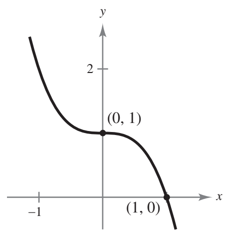
13. $y = x^3 + 2$

x	-2	-1	0	1	2
y	-6	1	2	3	10



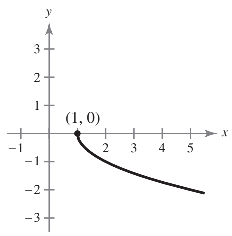
14. $y = 1 - x^3$

x	0	1	−1	2
y	1	0	2	−7



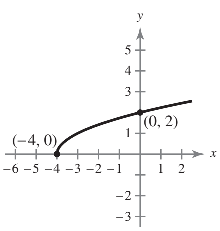
15. $y = -\sqrt{x - 1}$

x	1	2	3	4	5
y	0	−1	−1.41	−1.73	−2



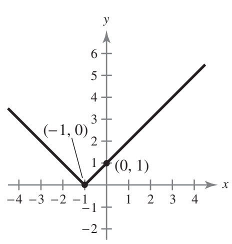
16. $y = \sqrt{x + 4}$

x	−4	−3	−2	−1	0
y	0	1	$\sqrt{2}$	$\sqrt{3}$	2



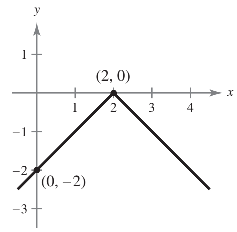
17. $y = |x + 1|$

x	−3	−2	−1	0	1
y	2	1	0	1	2



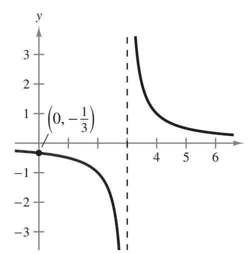
18. $y = -|x - 2|$

x	2	0	1	3	4
y	0	−2	−1	−1	−2



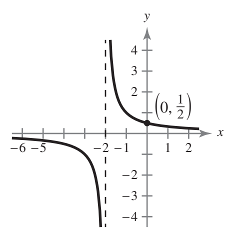
19. $y = \dfrac{1}{x - 3}$

x	−1	0	1	2	2.5	3.5	4	5	6
y	$-\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{2}$	−1	−2	2	1	$\frac{1}{2}$	$\frac{1}{3}$



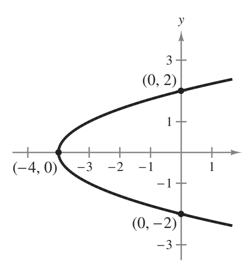
20. $y = \dfrac{1}{x + 2}$

x	−4	−3	−1	0	1	2
y	$-\frac{1}{2}$	−1	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$



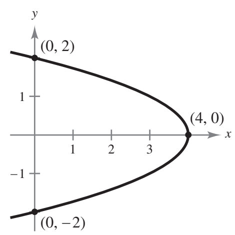
21. $x = y^2 - 4$

x	5	0	−3	−4
y	±3	±2	±1	0



22. $x = 4 - y^2$

x	0	3	4
y	± 2	± 1	0



23. Let $y = 0$. Then,

$$2x - (0) - 3 = 0$$
$$x = \tfrac{3}{2}.$$

Let $x = 0$. Then,

$$2(0) - y - 3 = 0$$
$$y = -3.$$

x-intercept: $\left(\tfrac{3}{2}, 0\right)$

y-intercept: $(0, -3)$

24. Let $y = 0$. Then,

$$4x - 3(0) - 6 = 0$$
$$4x = 6$$
$$x = \tfrac{3}{2}$$

Let $x = 0$. Then,

$$4(0) - 3y - 6 = 0$$
$$-3y = 6$$
$$y = -2$$

x-intercept: $\left(\tfrac{3}{2}, 0\right)$

y-intercept: $(0, -2)$

25. Let $y = 0$. Then,

$$0 = x^2 + x - 2$$
$$0 = (x + 2)(x - 1)$$
$$x = -2, 1.$$

Let $x = 0$. Then,

$$y = (0)^2 + (0) - 2$$
$$y = -2$$

x-intercepts: $(-2, 0), (1, 0)$

y-intercept: $(0, -2)$

26. Let $y = 0$. Then,

$$0 = x^2 - 4x + 3$$
$$0 = (x - 3)(x - 1)$$
$$x = 1, 3.$$

Let $x = 0$. Then,

$$y = (0)^2 - 4(0) + 3$$
$$y = 3.$$

x-intercepts: $(1, 0), (3, 0)$

y-intercept: $(0, 3)$

27. Let $y = 0$. Then,

$$0 = x^3 + 7x^2$$
$$0 = x^2(x + 7)$$
$$x^2 = 0 \rightarrow x = 0$$
$$x + 7 = 0 \rightarrow x = -7$$

Let $x = 0$. Then,

$$y = (0)^3 + 7(0)^2$$
$$y = 0$$

x-intercepts: $(0, 0), (-7, 0)$

y-intercept: $(0, 0)$

28. Let $y = 0$. Then,

$$0 = x^3 - 9x^2$$
$$0 = x^2(x - 9)$$
$$x^2 = 0 \rightarrow x = 0$$
$$x - 9 = 0 \rightarrow x = 9$$

Let $x = 0$. Then,

$$y = (0)^3 - 9(0)^2$$
$$y = 0$$

x-intercept: $(0, 0), (9, 0)$

y-intercept: $(0, 0)$

29. Let $y = 0$. Then,

$$0 = \frac{x^2 - 4}{x - 2}$$
$$0 = (x - 2)(x + 2)$$
$$x = \pm 2.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 - 4}{(0) - 2}$$
$$y = 2.$$

x-intercept: Because the equation is undefined when $x = 2$, the only x-intercept is $(-2, 0)$.

y-intercept: $(0, 2)$

30. Let $y = 0$. Then,

$$0 = \frac{x^2 + 3x}{2x}$$
$$0 = x(x + 3)$$
$$x = -3, 0.$$

Let $x = 0$. Then,

$$y = \frac{(0)^2 + 3(0)}{2(0)}$$
$$y = \text{undefined}.$$

x-intercept: Because the equation is undefined when $x = 0$, the only x-intercept is $(-3, 0)$.

y-intercept: Because the equation is undefined when $y = 0$, there is no y-intercept.

31. Let $y = 0$. Then,

$$x^2(0) - x^2 + 4(0) = 0$$
$$x^2 = 0$$
$$x = 0.$$

Let $x = 0$. Then,

$$(0)^2 y - (0)^2 + 4y = 0$$
$$y = 0.$$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

32. Let $y = 0$. Then,

$$2x^2(0) + 8(0) - x^2 = 1$$
$$x^2 = -1$$
$$x = \pm\sqrt{-1}.$$

Let $x = 0$. Then,

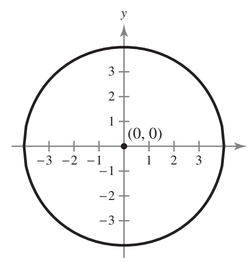
$$2(0)^2 y + 8y - (0)^2 = 1$$
$$y = \tfrac{1}{8}.$$

x-intercept: Because the equation has no real roots when $y = 0$, there is no x-intercept.

y-intercept: $\left(0, \tfrac{1}{8}\right)$

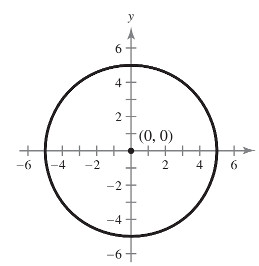
33. $(x - 0)^2 + (y - 0)^2 = 4^2$

$$x^2 + y^2 = 16$$



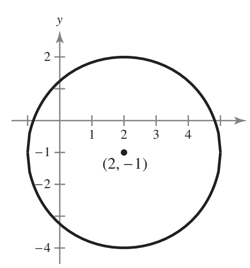
34. $(x - 0)^2 + (y - 0)^2 = 5^2$

$$x^2 + y^2 = 25$$



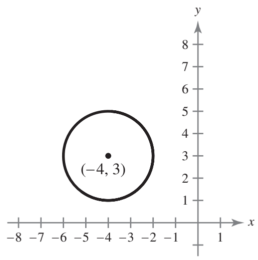
35. $(x - 2)^2 + (y - (-1))^2 = 3^2$

$$(x - 2)^2 + (y + 1)^2 = 9$$



36. $(x - (-4))^2 + (y - 3)^2 = 2^2$

$(x + 4)^2 + (y - 3)^2 = 4$



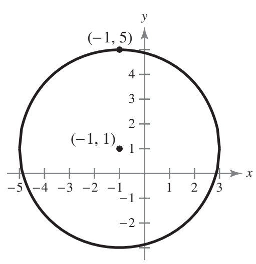
37. The radius is the distance between $(-1, 5)$ and $(-1, 1)$.

$$r = \sqrt{(-1 - (-1))^2 + (5 - 1)^2} = \sqrt{0^2 + 4^2} = \sqrt{16} = 4$$

Using the center $(-1, 1)$ and the radius $r = 4$:

$(x - (-1))^2 + (y - 1)^2 = 4^2$

$(x + 1)^2 + (y - 1)^2 = 16$



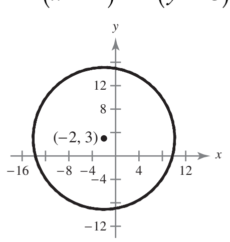
38. The radius is the distance between $(-2, 3)$ and $(5, -7)$.

$$r = \sqrt{(5 - (-2))^2 + (-7 - 3)^2} = \sqrt{7^2 + (-10)^2} = \sqrt{49 + 100} = \sqrt{149}$$

Using the center $(-2, 3)$ and the radius $r = \sqrt{149}$:

$(x - (-2))^2 + (y - 3)^2 = \left(\sqrt{149}\right)^2$

$(x + 2)^2 + (y - 3)^2 = 149$



39. The diameter is the distance between $(-6, -8)$ and $(6, 8)$.

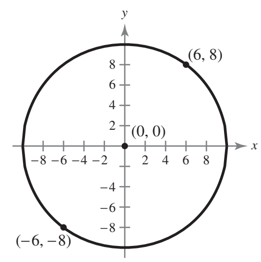
$$d = \sqrt{(6 - (-6))^2 + (8 - (-8))^2} = \sqrt{12^2 + 16^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

The radius is one-half the diameter: $r = \frac{20}{2} = 10$.

The center is the midpoint of the diameter: $\left(\frac{-6 + 6}{2}, \frac{-8 + 8}{2}\right) = (0, 0)$.

$(x - 0)^2 + (y - 0)^2 = 10^2$

$x^2 + y^2 = 100$



40. The diameter is the distance between $(0, -4)$ and $(6, 4)$.

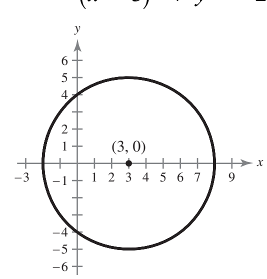
$$d = \sqrt{(6 - 0)^2 + (4 - (-4))^2} = \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

The radius is one-half the diameter: $r = \frac{10}{2} = 5$.

The center is the midpoint of the diameter: $\left(\frac{0 + 6}{2}, \frac{4 + 4}{2}\right) = (3, 0)$

$(x - 3)^2 + (y - 0)^2 = 5^2$

$(x - 3)^2 + y^2 = 25$



41. Set the two equations equal to each other.

$$-x + 2 = 2x - 1$$
$$-3x = -3$$
$$x = 1$$

Substitute $x = 1$ into one of the equations.

$$y = (-1) + 2 = 1$$

The point of intersection is $(1, 1)$.

42. Set the two equations equal to each other.

$$-x + 7 = \tfrac{3}{2}x - 8$$
$$-2x + 14 = 3x - 16$$
$$-5x = -30$$
$$x = 6$$

Substitute $x = 6$ into one of the equations.

$$y = (-6) + 7 = 1$$

The point of intersection is $(6, 1)$.

43. Set the two equations equal to each other.

$$-x^2 + 15 = 3x + 11$$
$$-x^2 - 3x + 4 = 0$$
$$x^2 + 3x - 4 = 0$$
$$(x + 4)(x - 1) = 0$$
$$x + 4 = 0 \qquad x - 1 = 0$$
$$x = -4 \qquad x = 1$$

Substitute $x = -4$:

$$y = -(-4)^2 + 15$$
$$y = -16 + 15$$
$$y = -1$$

Substitute $x = 1$:

$$y = -(1)^2 + 15$$
$$y = -1 + 15$$
$$y = 14$$

The points of intersection are $(-4, -1)$ and $(1, 14)$.

44. Set the two equations equal to each other.

$$x^2 - 5 = x + 1$$
$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \qquad x + 2 = 0$$
$$x = 3 \qquad x = -2$$

Substitute $x = 3$:

$$y = (3)^2 - 5$$
$$y = 4$$

Substitute $x = -2$:

$$y = (-2)^2 - 5$$
$$y = -1$$

The points of intersection are $(3, 4)$ and $(-2, -1)$.

45. Set the two equations equal to each other.

$$x^3 = 2x$$
$$x^3 - 2x = 0$$
$$x(x^2 - 2) = 0$$
$$x = 0, \pm\sqrt{2}$$

Substitute $x = 0$:

$$y = 2(0)$$
$$y = 0$$

Substitute $x = \sqrt{2}$:

$$y = 2(\sqrt{2})$$
$$y = 2\sqrt{2}$$

Substitute $x = -\sqrt{2}$:

$$y = 2(-\sqrt{2})$$
$$y = -2\sqrt{2}$$

The points of intersection are $(0, 0)$, $(-\sqrt{2}, -2\sqrt{2})$, and $(\sqrt{2}, 2\sqrt{2})$.

46. Set the two equations equal to each other.

$$\sqrt{x} = x$$
$$x = x^2$$
$$0 = x(x - 1)$$
$$x = 0, 1$$

Substitute $x = 0$:

$$y = 0$$

Substitute $x = 1$:

$$y = 1$$

The points of intersection are (0, 0) and (1, 1).

47. Set the two equations equal to each other.

$$x^4 - 2x^2 + 1 = 1 - x^2$$
$$x^4 - x^2 = 0$$
$$x^2(x + 1)(x - 1) = 0$$
$$x = 0, \pm 1$$

Substitute $x = 0$:

$$y = 1 - (0)^2$$
$$y = 1$$

Substitute $x = 1$:

$$y = 1 - (1)^2$$
$$y = 0$$

Substitute $x = -1$:

$$y = 1 - (-1)^2$$
$$y = 1 - 1$$
$$y = 0$$

The points of intersection are $(-1, 0)$, $(0, 1)$, and $(1, 0)$.

48. Set the two equations equal to each other.

$$x^3 - 2x^2 + x - 1 = -x^2 + 3x - 1$$
$$x^3 - x^2 - 2x = 0$$
$$x(x + 1)(x - 2) = 0$$
$$x = 0, -1, 2$$

Substitute $x = 0$:

$$y = -(0)^2 + 3(0) - 1$$
$$y = 0 + 0 - 1$$
$$y = -1$$

Substitute $x = -1$:

$$y = -(-1)^2 + 3(-1) - 1$$
$$y = -1 - 3 - 1$$
$$y = -5$$

Substitute $x = 2$:

$$y = -(2)^2 + 3(2) - 1$$
$$y = -4 + 6 - 1$$
$$y = 1$$

The points of intersection are $(0, -1)$, $(-1, -5)$, and $(2, 1)$.

49. To find the break-even point, set $R = C$.

$$1.55x = 0.85x + 35{,}000$$
$$0.7x = 35{,}000$$
$$x = \frac{35{,}000}{0.7} = 50{,}000 \text{ units}$$

50. To find the break-even point, set $R = C$.

$$35x = 6x + 500{,}000$$
$$29x = 500{,}000$$
$$x = \frac{500{,}000}{29} \approx 17{,}242 \text{ units}$$

51. To find the break-even point, set $R = C$.

$$9950x = 8650x + 250{,}000$$
$$1300x = 250{,}000$$
$$x = \frac{250{,}000}{1300} \approx 193 \text{ units}$$

52. To find the break-even point, set $R = C$.

$$4.9x = 2.5x + 10{,}000$$
$$2.4x = 10{,}000$$
$$x = \frac{10{,}000}{2.4} \approx 4167 \text{ units}$$

53. To find the break-even point, set $R = C$.

$$10x = 6x + 5000$$
$$4x = 5000$$
$$x = \frac{5000}{4} \approx 1250 \text{ units}$$

54. To find the break-even point, set $R = C$.

$$200x = 130x + 12{,}600$$
$$70x = 12{,}600$$
$$x = \frac{12{,}600}{70} \approx 180 \text{ units}$$

55. (a) $C = 11.5x + 21{,}000$

$R = 19.90x$

(b)

$$C = R$$
$$11.5x + 21{,}000 = 19.90x$$
$$21{,}000 = 8.4x$$
$$x = 2500 \text{ units}$$

(c)

$$P = R - C$$
$$1000 = 19.9x - (11.5x + 21{,}000)$$
$$22{,}000 = 8.4x$$
$$x \approx 2619 \text{ units}$$

So, 2619 units would yield a profit of \$1000.

56. (a) The cost C_g to drive x miles is the cost of the car itself plus the cost of gasoline per mile, which is the cost of gasoline per gallon divided by the number of gallons per mile.

$$C_g = 33{,}500 + \frac{2.759}{31}x$$

Similarly, the cost C_h to drive x miles is the cost of the car itself plus the cost of gasoline per mile.

$$C_h = 36{,}775 + \frac{2.759}{39}x$$

(b) To find the break-even point, set the cost equations equal to each other.

$$33{,}500 + \frac{2.759}{31}x = 36{,}775 + \frac{2.759x}{39}$$

Multiply both sides of the equation by $(31)(39)$.

$$40{,}501{,}000 + 107.601x = 44{,}460{,}975 + 85.529x$$
$$22.072x = 3{,}959{,}975$$
$$x = \frac{3{,}959{,}975}{22.072} \approx 179{,}412 \text{ mi}$$

57.

$$205 - 4x = 135 + 3x$$
$$70 = 7x$$
$$10 = x$$

Equilibrium point $(x, p) = (10, 165)$

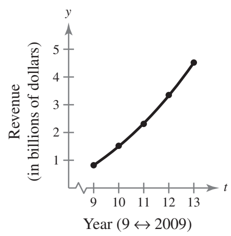
58.

$$190 - 15x = 75 + 8x$$
$$115 = 23x$$
$$x = 5$$

Equilibrium point $(x, p) = (5, 115)$

59. (a)

Year	2009	2010	2011	2012	2013
Revenue	0.82	1.52	2.31	3.35	4.52
Model	0.82	1.50	2.33	3.33	4.52



The model fits the data well.

(b) Let $t = 18$ (2018).

$$y = 0.00333(18)^3 - 0.0250(18)^2 + 0.252(18) - 1.85 \approx \$14.0 \text{ billion}$$

60. (a) If 10,000 units are sold, the company breaks even.

(b) If less than 10,000 units are sold, the company loses money.

(c) If more than 10,000 units are sold, the company makes a profit.

61. (a)

Year	2008	2009	2010	2011	2012	2016
Degrees	747	793	854	931	1024	1550

(b) Answers will vary.

(c) Let $t = 20$ (2020).

$$y = 7.79(20)^2 - 86.6(20) + 941$$
$$= 2325 \text{ degrees}$$

The prediction is valid because the number of associate's degrees should keep increasing over time.

62. (a) and (b)

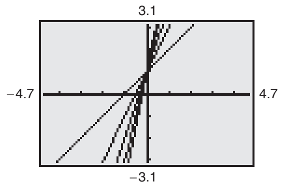
Year	2009	2010	2011	2012	2013
Transplants (model)	2213.79	2323.0	2336.79	2369.16	2534.11
Transplants (actual)	2211	2332	2322	2378	2531

(c) For 2019, let $t = 19$.

$$y = 19.000(19)^3 - 617.71(19)^2 + 6696.7(19) - 21{,}873 \approx 12{,}692$$

The prediction seems high. Answers will vary.

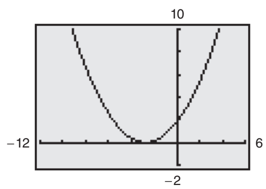
63.



The greater the value of c, the steeper the line.

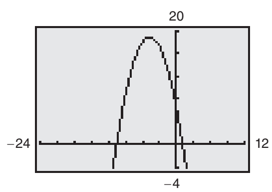
64. If C and R represent the cost and revenue for a business, the break-even point is that value of x for which $C = R$. For example, if $C = 100{,}000 + 10x$ and $R = 20x$, then the break-even point is $x = 10{,}000$ units.

65.



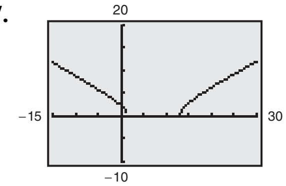
Intercepts: $(-2.75,\ 0), (0, 1.815)$

66.



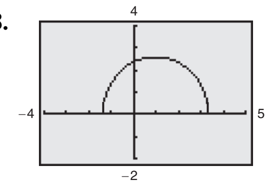
Intercepts: $(0,\ 6.25)$, $(1.0539,\ 0)$, $(-10.5896,\ 0)$

67.



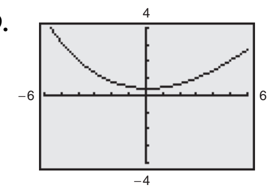
Intercepts: $(1.4780,\ 0)$, $(12.8553,\ 0)$, $(0,\ 2.3875)$

68.



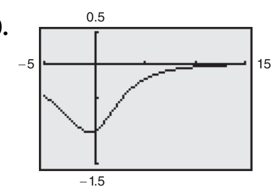
Intercepts: $(3.3256,\ 0)$, $(-1.3917,\ 0)$, $(0,\ 2.3664)$

69.



Intercept: $\left(0,\ \frac{5}{12}\right) \approx (0,\ 0.4167)$

70.



Intercepts: $(0,\ -1)$, $(13.25,\ 0)$

71. Answers will vary.

Section 1.3 Lines in the Plane and Slope

Skills Warm Up

1. $\dfrac{5 - (-2)}{-3 - 4} = \dfrac{7}{-7} = -1$

2. $\dfrac{-4 - (-10)}{7 - 5} = \dfrac{6}{2} = 3$

3. $-\dfrac{1}{m},\ m = -3$

$-\dfrac{1}{-3} = \dfrac{1}{3}$

4. $-\dfrac{1}{m},\ m = \dfrac{6}{7}$

$-\dfrac{1}{\frac{6}{7}} = -\dfrac{7}{6}$

5. $-4x + y = 7$

$y = 4x + 7$

6. $3x - y = 7$

$-y = 7 - 3x$

$y = 3x - 7$

Skills Warm Up —*continued*—

7. $y - 2 = 3(x - 4)$

$y = 3(x - 4) + 2$

$y = 3x - 12 + 2$

$y = 3x - 10$

8. $y - (-5) = -1[x - (-2)]$

$y + 5 = -x - 2$

$y = -x - 7$

9. $y - (-3) = \dfrac{4 - (-2)}{11 - 3}(x - 12)$

$y + 3 = \dfrac{6}{8}(x - 12)$

$y + 3 = \dfrac{3}{4}(x - 12)$

$y + 3 = \dfrac{3}{4}x - 9$

$y = \dfrac{3}{4}x - 12$

10. $y - 1 = \dfrac{-3 - 1}{-7 - (-1)}[x - (-1)]$

$y - 1 = \dfrac{-4}{-6}(x + 1)$

$y - 1 = \dfrac{2}{3}(x + 1)$

$y - 1 = \dfrac{2}{3}x + \dfrac{2}{3}$

$y = \dfrac{2}{3}x + \dfrac{5}{3}$

1. The slope is $m = 1$ because the line rises one unit vertically for each unit the line moves to the right.

2. The slope is 2 because the line rises two units vertically for each unit the line moves to the right.

3. The slope is $m = 0$ because the line is horizontal.

4. The slope is -1 because the line falls one unit vertically for each unit the line moves to the right.

5. $y = x + 7$

So, the slope is $m = 1$, and the y-intercept is $(0, 7)$.

6. $y = 4x + 3$

So, the slope is $m = 4$, and the y-intercept is $(0, 3)$.

7. $5x + y = 20$

$y = -5x + 20$

So, the slope is $m = -5$, and the y-intercept is $(0, 20)$.

8. $2x + y = 40$

$y = -2x + 40$

So, the slope is $m = -2$, and the y-intercept is $(0, 40)$.

9. $7x + 6y = 30$

$y = -\frac{7}{6}x + 5$

So, the slope is $m = -\frac{7}{6}$, and the y-intercept is $(0, 5)$.

10. $8x + 3y = 12$

$3y = -8x + 12$

$y = -\frac{8}{3}x + 4$

So, the slope is $m = -\frac{8}{3}$, and the y-intercept is $(0, 4)$.

11. $3x - y = 15$

$y = 3x - 15$

So, the slope is $m = 3$, and the y-intercept is $(0, -15)$.

12. $2x - 3y = 24$

$y = \frac{2}{3}x - 8$

So, the slope is $m = \frac{2}{3}$, and the y-intercept is $(0, -8)$.

13. $x = 4$

Because the line is vertical, the slope is undefined. There is no y-intercept.

14. $x + 5 = 0$

$x = -5$

Because the line is vertical, the slope is undefined. There is no y-intercept.

15. $y - 9 = 0$

$y = 9$

So, the slope is $m = 0$, and the y-intercept is $(0, 9)$.

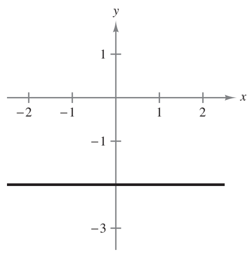
16. $y + 1 = 0$

$y = -1$

So, the slope is $m = 0$, and the y-intercept is $(0, -1)$.

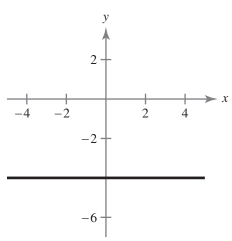
17. $y = -2$

x	-2	-1	0	1
y	-2	-2	-2	-2



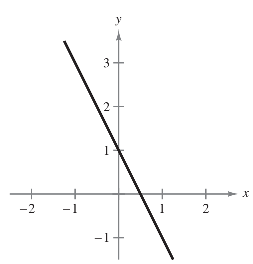
18. $y = -4$

x	-4	-2	0	2
y	-4	-4	-4	-4



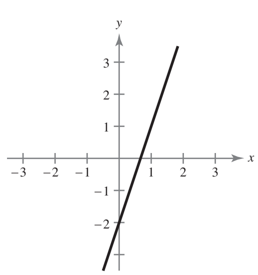
19. $y = -2x + 1$

x	-1	0	1	2
y	3	1	-1	-3



20. $y = 3x - 2$

x	-1	0	1	2
y	-5	-2	1	4

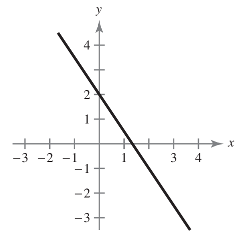


21. $3x + 2y = 4$

$2y = -3x + 4$

$y = -\frac{3}{2}x + 2$

x	-4	-2	0	2	4
y	8	5	2	-1	-4

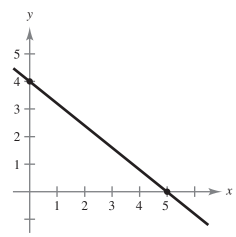


22. $4x + 5y = 20$

$5y = -4x + 20$

$y = -\frac{4}{5}x + 4$

x	0	2	4	5
y	4	$\frac{12}{5}$	$\frac{4}{5}$	0

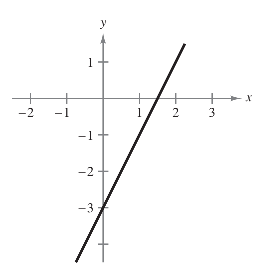


23. $2x - y - 3 = 0$

$-y = -2x + 3$

$y = 2x - 3$

x	-1	0	1	2
y	-5	-3	-1	1

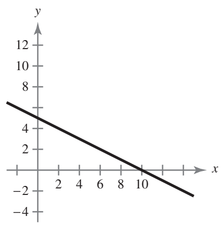


24. $x + 2y + 10 = 0$

$$2y = -x + 10$$

$$y = -\tfrac{1}{2}x + 5$$

x	-4	-2	0	2	4
y	7	6	5	4	3

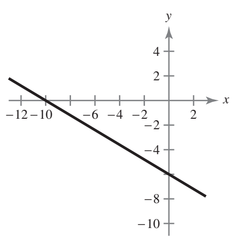


25. $3x + 5y + 30 = 0$

$$5y = -3x - 30$$

$$y = -\tfrac{3}{5}x - 6$$

x	-10	-5	0	5	10
y	0	-3	-6	-9	-12

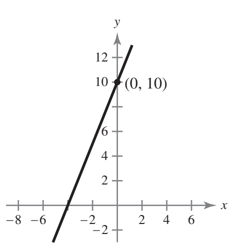


26. $-5x + 2y - 20 = 0$

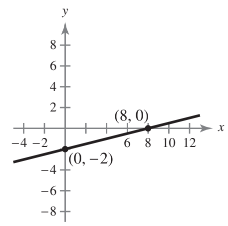
$$2y = 5x + 20$$

$$y = \tfrac{5}{2}x + 10$$

x	-4	-2	0	2
y	0	5	10	15

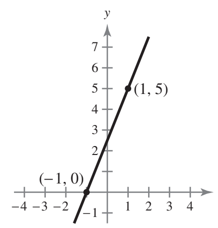


27.



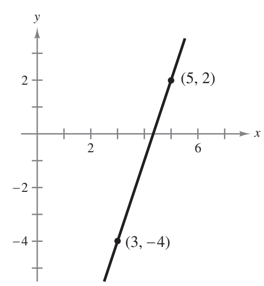
The slope is $m = \dfrac{0 - (-2)}{8 - 0} = \dfrac{2}{8} = \dfrac{1}{4}$.

28.



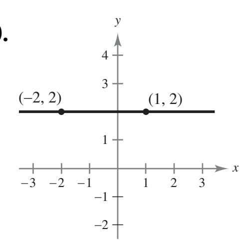
The slope is $m = \dfrac{5 - 0}{1 - (-1)} = \dfrac{5}{2}$.

29.



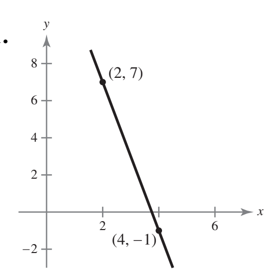
The slope is $m = \dfrac{2 - (-4)}{5 - 3} = 3$.

30.



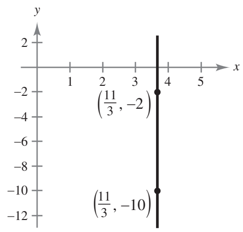
The slope is $m = \dfrac{2 - 2}{1 - (-2)} = 0$.

31.



The slope of $m = \dfrac{7 - (-1)}{2 - 4} = \dfrac{8}{-2} = -4$.

32.

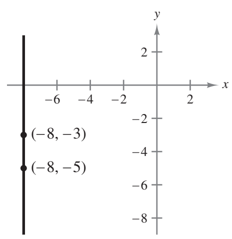


The slope is $m = \dfrac{-10 - (-2)}{\dfrac{11}{3} - \dfrac{11}{3}} = \dfrac{-8}{0}$,

which is undefined.

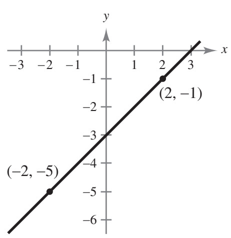
So, the line is vertical.

33.



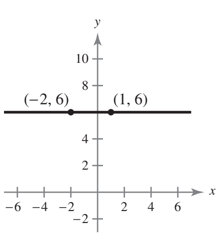
The slope is undefined because $m = \dfrac{-5 - (-3)}{-8 - (-8)}$ and division by zero is undefined. So, the line is vertical.

34.



The slope is $m = \dfrac{-1 - (-5)}{2 - (-2)} = \dfrac{4}{4} = 1.$

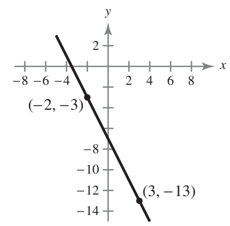
35.



The slope is $m = \dfrac{6 - 6}{1 - (-2)} = \dfrac{0}{3} = 0.$

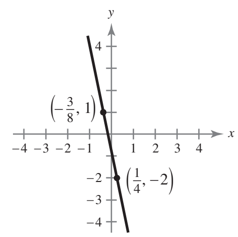
So, the line is horizontal.

36.



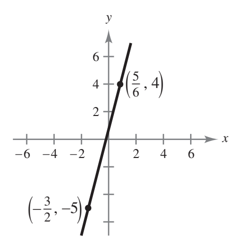
The slope is $m = \dfrac{-3 - (-13)}{-2 - 3} = \dfrac{10}{-5} = -2.$

37.



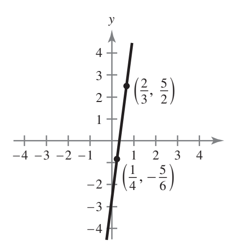
The slope is $m = \dfrac{1 - (-2)}{-\frac{3}{8} - \frac{1}{4}} = \dfrac{3}{-\frac{5}{8}} = -\dfrac{24}{5}.$

38.



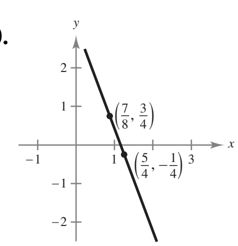
The slope is $m = \dfrac{4 + 5}{(5/6) + (3/2)} = \dfrac{27}{7}.$

39.



The slope is $m = \dfrac{\frac{5}{2} - \left(-\frac{5}{6}\right)}{\frac{2}{3} - \frac{1}{4}} = \dfrac{\frac{10}{3}}{\frac{5}{12}} = \dfrac{10}{3} \cdot \dfrac{12}{5} = 8.$

40.



The slope is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}.$

41. The equation of this horizontal line is $y = 1$. So, three additional points are $(0, 1)$, $(1, 1)$, and $(3, 1)$.

42. The equation of this horizontal line is $y = -3$. So, three additional points are $(0, -3)$, $(1, -3)$, and $(2, -3)$.

43. The equation of the line is

$$y - 7 = -3(x - 1)$$
$$y = -3x + 10.$$

So, three additional points are $(0, 10)$, $(2, 4)$, and $(3, 1)$.

44. The equation of this line is

$$y + 2 = 2(x - 7)$$
$$y = 2x - 16.$$

So, three additional points are $(0, -16)$, $(1, -14)$, and $(2, -12)$.

45. The equation of this line is

$$y + 4 = \frac{2}{3}(x - 6)$$
$$y = \frac{2}{3}x - 8.$$

So, three additional points are $(3, -6)$, $(9, -2)$, and $(12, 0)$.

46. The equation of this line is

$$y + 6 = -\tfrac{1}{2}(x + 1)$$
$$y = -\tfrac{1}{2}x - \tfrac{13}{2}.$$

So, three additional points are $(1, -7)$, $(3, -8)$, and $(5, -9)$.

47. The equation of this vertical line is $x = -8$. So, three additional points are $(-8, 0)$, $(-8, 2)$, and $(-8, 3)$.

48. The equation of this vertical line is $x = -3$. So, three additional points are $(-3, 0)$, $(-3, 1)$, and $(-3, 2)$.

49. The slope of the line joining $(-2, 1)$ and $(-1, 0)$ is

$$\frac{1 - 0}{-2 - (-1)} = \frac{1}{-1} = -1.$$

The slope of the line joining $(-1, 0)$ and $(2, -2)$ is

$$\frac{0 - (-2)}{-1 - 2} = \frac{2}{-3} = -\frac{2}{3}.$$

Because the slopes are different, the points are not collinear.

50. The slope of the line joining $(-5, 11)$ and $(0, 4)$ is

$$\frac{11 - 4}{-5 - 0} = \frac{7}{-5} = -\frac{7}{5}.$$

The slope of the line joining $(0, 4)$ and $(7, -6)$ is

$$\frac{4 - (-6)}{0 - 7} = -\frac{10}{7}.$$

Because the slopes are different, the points are not collinear.

51. The slope of the line joining $(2, 7)$ and $(-2, -1)$ is

$$\frac{-1 - 7}{-2 - 2} = 2.$$

The slope of the line joining $(0, 3)$ and $(-2, -1)$ is

$$\frac{-1 - 3}{-2 - 0} = 2.$$

Because the slopes are equal and both lines pass through $(-2, -1)$, the three points are collinear.

52. The slope of the line joining $(4, 1)$ and $(-2, -2)$ is

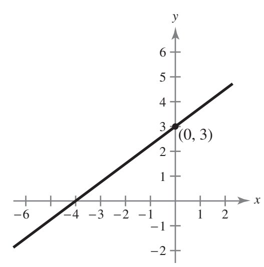
$$\frac{-2 - 1}{-2 - 4} = \frac{1}{2}.$$

The slope of the line joining $(8, 3)$ and $(-2, -2)$ is

$$\frac{-2 - 3}{-2 - 8} = \frac{1}{2}.$$

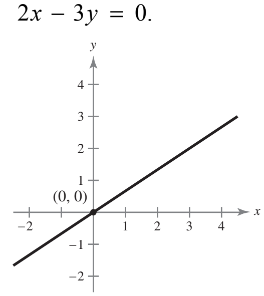
Because the slopes are equal and both lines pass through $(-2, -2)$, the three points are collinear.

53. Using the slope-intercept form, we have $y = \frac{3}{4}x + 3$.

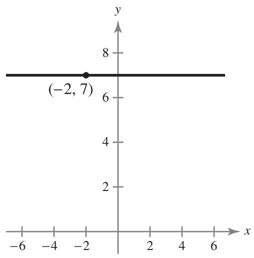


54. Using the slope-intercept form, we have

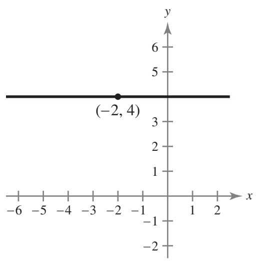
$$y = \tfrac{2}{3}x + 0$$
$$2x - 3y = 0.$$



55. Because the slope is 0, the line is horizontal and its equation is $y = 7$.

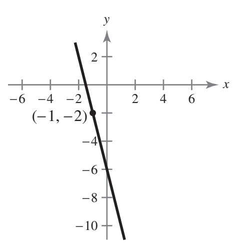


56. Because the slope is 0, the line is horizontal and its equation is $y = 4$.



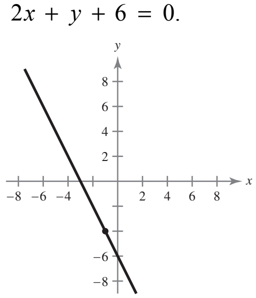
57. Using the point-slope form, you have

$$y - (-2) = -4(x - (-1))$$
$$y + 2 = -4(x + 1)$$
$$y + 2 = -4x - 4$$
$$y = -4x - 6$$
$$4x + y + 6 = 0.$$



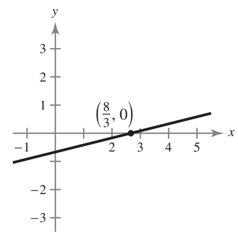
58. Using the point-slope form, you have

$$y + 4 = -2(x + 1)$$
$$y = -2x - 6$$
$$2x + y + 6 = 0.$$



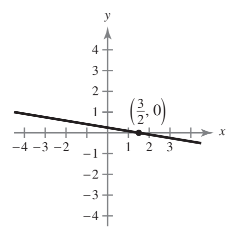
59. Using the point-slope form, you have

$$y - 0 = \tfrac{1}{4}\left(x - \tfrac{8}{3}\right)$$
$$y = \tfrac{1}{4}x - \tfrac{2}{3}$$
$$3x - 12y - 8 = 0.$$



60. Using the point-slope form, you have

$$y - 0 = -\tfrac{1}{6}\left(x - \tfrac{3}{2}\right)$$
$$y = -\tfrac{1}{6}x + \tfrac{1}{4}$$
$$2x + 12y - 3 = 0.$$

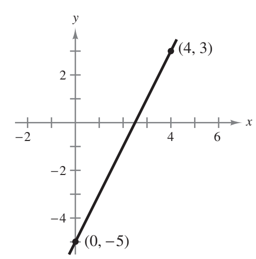


61. The slope of the line is

$$m = \frac{3 - (-5)}{4 - 0} = 2.$$

Using the point-slope form, you have

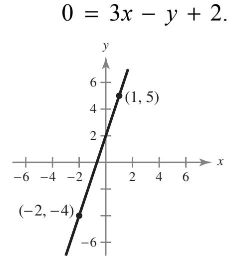
$$y + 5 = 2(x - 0)$$
$$y = 2x - 5$$
$$0 = 2x - y - 5.$$



62. The slope of the line is $m = \dfrac{5 - (-4)}{1 - (-2)} = \dfrac{9}{3} = 3$.

Using the point-slope form, you have

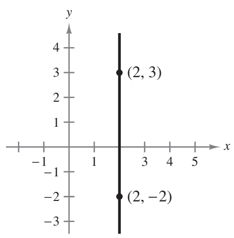
$$y - 5 = 3(x - 1)$$
$$y - 5 = 3x - 3$$
$$0 = 3x - y + 2.$$



63. The slope of the line is $m = \dfrac{-2 - 3}{2 - 2} = \text{undefined}.$

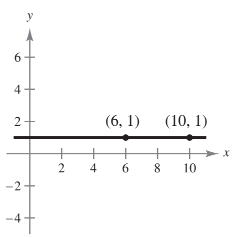
So, the line is vertical, and its equation is

$$x = 2$$
$$x - 2 = 0.$$



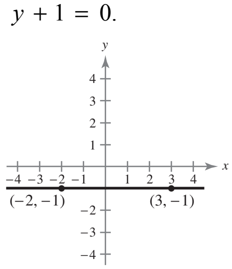
64. The slope of the line is $m = \dfrac{1 - 1}{10 - 6} = 0.$ So, the line is horizontal, and its equation is

$$y = 1$$
$$y - 1 = 0.$$



65. The slope of the line is $m = \dfrac{-1 - (-1)}{-2 - 3} = 0.$ So, the line is horizontal, and its equation is

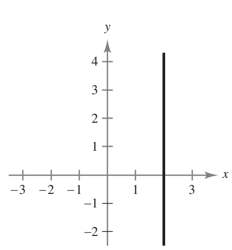
$$y = -1$$
$$y + 1 = 0.$$



66. The slope of the line is $m = \dfrac{-10 - 5}{2 - 2} = \text{undefined}.$

So, the line is vertical, and its equation is

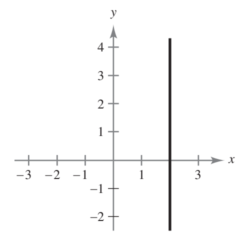
$$x = 2$$
$$x - 2 = 0.$$



67. The slope of the line is $m = \dfrac{8 - 4}{1/2 + 1/2} = 4.$

Using the point-slope form, you have

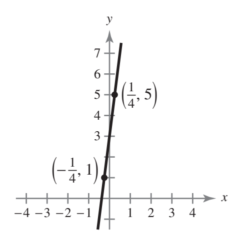
$$y - 8 = 4\left(x - \frac{1}{2}\right)$$
$$y = 4x + 6$$
$$0 = 4x - y + 6.$$



68. The slope is $m = \dfrac{5 - 1}{\frac{1}{4} - \left(-\frac{1}{4}\right)} = \dfrac{4}{\frac{1}{2}} = 8.$

Using the point-slope form, you have

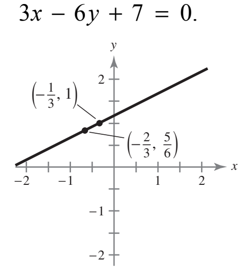
$$y - 1 = 8\left(x - \left(-\frac{1}{4}\right)\right)$$
$$y - 1 = 8x + 2$$
$$y = 8x + 3$$
$$8x - y + 3 = 0.$$



69. The slope of the line is $m = \dfrac{1 - 5/6}{(-1/3) + 2/3} = \dfrac{1}{2}.$

Using the point-slope form, you have

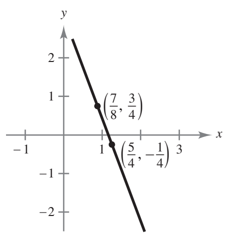
$$y - 1 = \frac{1}{2}\left(x + \frac{1}{3}\right)$$
$$y = \frac{1}{2}x + \frac{7}{6}$$
$$3x - 6y + 7 = 0.$$



70. The slope of the line is $m = \dfrac{(-1/4) - (3/4)}{(5/4) - (7/8)} = -\dfrac{8}{3}$.

Using the point-slope form, you have

$$y - \frac{3}{4} = -\frac{8}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{8}{3}x + \frac{7}{3}$$
$$12y - 9 = -32x + 28$$
$$32x + 12y - 37 = 0.$$



71. Because the line is vertical, it has an undefined slope, and its equation is

$$x = 3$$
$$x - 3 = 0.$$

72. Because the line is horizontal, it has a slope of $m = 0$, and its equation is

$$y = 0x + (-5)$$
$$y = -5.$$

73. Because the line is parallel to all horizontal lines, it has a slope of $m = 0$, and its equation is

$$y = -10.$$

74. Because the line is parallel to all vertical lines, it has an undefined slope, and its equation is

$$x = -5.$$

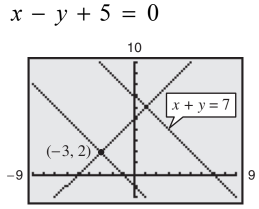
75. Given line: $y = -x + 7$, $m = -1$

(a) Parallel: $m_1 = -1$

$$y - 2 = -1(x + 3)$$
$$x + y + 1 = 0$$

(b) Perpendicular: $m_2 = 1$

$$y - 2 = 1(x + 3)$$
$$x - y + 5 = 0$$



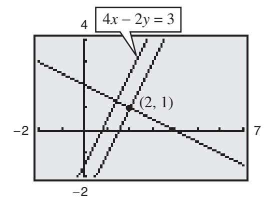
76. Given line: $y = 2x - \frac{3}{2}$, $m = 2$

(a) Parallel: $m_1 = 2$

$$y - 1 = 2(x - 2)$$
$$0 = 2x - y - 3$$

(b) Perpendicular: $m_2 = -\frac{1}{2}$

$$y - 1 = -\tfrac{1}{2}(x - 2)$$
$$2y - 2 = -x + 2$$
$$x + 2y - 4 = 0$$



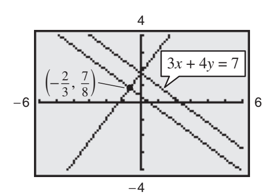
77. Given line: $y = -\frac{3}{4}x + \frac{7}{4}$, $m = -\frac{3}{4}$

(a) Parallel: $m_1 = -\frac{3}{4}$

$$y - \tfrac{7}{8} = -\tfrac{3}{4}\left(x + \tfrac{2}{3}\right) = -\tfrac{3}{4}x - \tfrac{1}{2}$$
$$8y - 7 = -6x - 4$$
$$6x + 8y - 3 = 0$$

(b) Perpendicular: $m_2 = \frac{4}{3}$

$$y - \tfrac{7}{8} = \tfrac{4}{3}\left(x + \tfrac{2}{3}\right) = \tfrac{4}{3}x + \tfrac{8}{9}$$
$$72y - 63 = 96x + 64$$
$$96x - 72y + 127 = 0$$



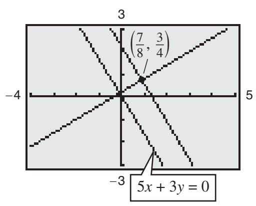
78. Given line: $y = -\frac{5}{3}x,\ m = -\frac{5}{3}$

(a) Parallel: $m_1 = -\frac{5}{3}$

$$y - \frac{3}{4} = -\frac{5}{3}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = -\frac{5}{3}x + \frac{35}{24}$$
$$24y - 18 = -40x + 35$$
$$40x + 24y - 53 = 0$$

(b) Perpendicular: $m_2 = \frac{3}{5}$

$$y - \frac{3}{4} = \frac{3}{5}\left(x - \frac{7}{8}\right)$$
$$y - \frac{3}{4} = \frac{3}{5}x - \frac{21}{40}$$
$$40y - 30 = 24x - 21$$
$$0 = 24x - 40y + 9$$



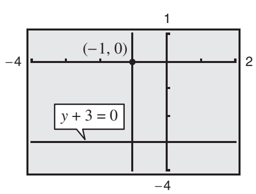
79. Given line: $y = -3$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0$

$$y - 0 = 0(x + 1)$$
$$y = 0$$

(b) Perpendicular: m_2 is undefined

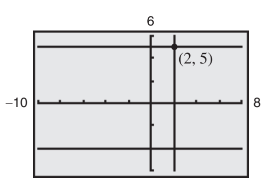
$$x = -1$$



80. Given line: $y + 4 = 0$ is horizontal, $m = 0$

(a) Parallel: $m_1 = 0,\ y - 5 = 0(x - 2),\ y = 5$

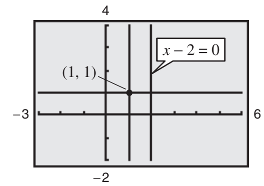
(b) Perpendicular: m_2 is undefined, $x = 2$



81. Given line: $x - 2 = 0$ is vertical, m is undefined

(a) Parallel: m_1 is undefined, $x = 1$

(b) Perpendicular: $m_2 = 0,\ y - 1 = 0(x - 1),\ y = 1$

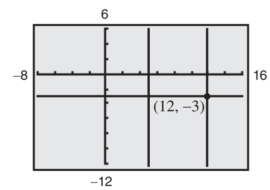


82. Given line: $x - 5 = 0$ is vertical, m is undefined.

(a) Parallel: m_1 is undefined, $x = 12$.

(b) Perpendicular:

$m_2 = 0,\ y + 3 = 0(x - 12),\ y = -3$



83. (a) The average salary increased the most from 2008 to 2009 and increased the least from 2010 to 2011.

(b) 2008: $(8, 92{,}920)$ and 2013: $(13, 100{,}600)$

$$m = \frac{100{,}600 - 92{,}920}{13 - 8} = \frac{7680}{5} = \$1536/\text{yr}$$

(c) The average salary increased $1536 per year over the 5 years between 2008 and 2013.

84. (a) The revenue increased the greatest from 2010 to 2011 and increased the least from 2011 to 2012.

(b) 2009: $(9, 123.0)$ and 2013: $(13, 128.8)$

$$m = \frac{128.8 - 123.0}{13 - 9} = \frac{5.8}{4} = \$1.45\ \text{billion/yr}$$

(c) From 2009 to 2013, the revenue for AT&T increased $1.45 billion per year.

85. $\frac{6}{100} = \frac{x}{200}$

$12 = x$

Since the grade of the road is $\frac{6}{100}$, if you drive 200 feet, the vertical rise in the road will be 12 feet.

86. (a) $(0, 32), (100, 212)$

$$F - 32 = \frac{212 - 32}{100 - 0}(C - 0)$$

$$F = 1.8C + 32 = \frac{9}{5}C + 32$$

or

$$C = \frac{5}{9}(F - 32)$$

(b) Use $C = \frac{5}{9}(F - 32)$. If $F = 102.2°\text{F}$, then

$$C = \frac{5}{9}(102.2 - 32) = 39°\text{C}.$$

(c) Use $C = \frac{5}{9}(F - 32)$. If $F = 76°\text{F}$, then

$$C = \frac{5}{9}(76 - 32) = 24.4°\text{C}.$$

87. (a) 2009: $(9, 5655)$ and 2013: $(13, 5743)$

$$m = \frac{5743 - 5655}{13 - 9} = \frac{88}{4} = 22$$

$$y - y_1 = m(t - t_1)$$

$$y - 5655 = 22(t - 9)$$

$$y - 5655 = 22t - 198$$

$$y = 22t + 5457$$

The slope is 22.0 and indicates that the population increases 22 thousand per year from 2009 to 2013.

(b) Let $t = 11$.

$$y = 22(11) + 5457$$

$$y = 5699$$

The population was 5699 thousand or 5,699,000 in 2011.

(c) The actual population in 2011 was 5,709,000.

The model's estimate was very close to the actual population.

(d) The model could possibly be used to predict the population in 2018 if the population continues to grow at the same linear rate.

88. (a) 2008: $(8, 12{,}430)$ and 2013: $(13, 14{,}167)$

$$m = \frac{14{,}167 - 12{,}430}{13 - 8} = \frac{1737}{5} = 347.4$$

$$y - y_1 = m(t - t_1)$$

$$y - 12{,}430 = 347.4(t - 8)$$

$$y - 12{,}430 = 347.4t - 2779.2$$

$$y = 347.4t + 9650.8$$

The slope is 347.4 and indicates that the personal income increases $347.3 billion per year from 2008 to 2013.

(b) Let $t = 11$.

$$y = 347.4(11) + 9650.8$$

$$y = 13{,}472.2$$

The personal income was $13,472.2 billion in 2011.

Let $t = 14$.

$$y = 347.4(14) + 9650.8$$

$$y = 14{,}514.4$$

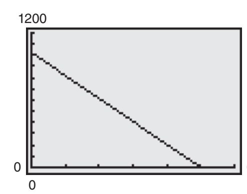
The personal income was $14,514.4 billion in 2014.

(c) The actual personal income was $13,202.0 billion in 2011 and $14,728.6 billion in 2014.

The model's estimates were very close to the actual personal incomes in 2011 and 2014.

89. (a) The equipment depreciates $\frac{1025}{5} = \$205$ per year, so the value is $y = 1025 - 205t$, where $0 \le t \le 5$.

(b)



(c) When $t = 3$, the value is $410.00.

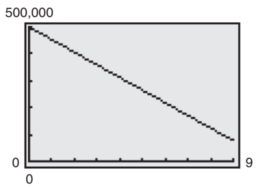
(d) The value is $600 when $t = 2.07$ years.

90. (a) The slope is

$$\frac{77{,}000 - 500{,}000}{9} = \frac{-423{,}000}{9} = -47{,}000.$$

The equipment depreciates \$47,000 per year, so the value is $y = 500{,}000 - 47{,}000t$, where $0 \le t \le 9$.

(b)



(c) When $t = 5$, the value is

$y = 500{,}000 - 47{,}000(5) = \$265{,}000.$

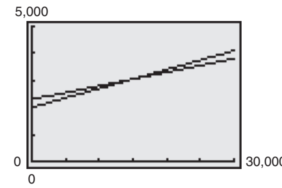
(d) The value is \$160,000 when

$$160{,}000 = 500{,}000 - 47{,}000t$$
$$47{,}000t = 340{,}000$$
$$t \approx 7.2 \text{ years.}$$

91. (a) Current wage: $W_c = 0.07s + 2000$

New offer wage: $W_N = 0.05s + 2300$

(b)



The lines intersect at $(15{,}000, 3050)$. If you sell \$15,000, then both jobs would yield wages of \$3050.

(c) No. Your current job would yield wages of \$3400 as compared to the new job, which would yield wages of \$3300 if your sales are \$20,000.

92. (a) Matches (ii); $y = -10x + 100$.

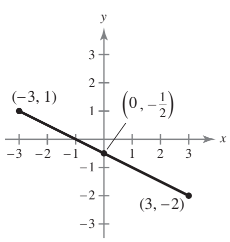
(b) Matches (iii); $y = 1.50x + 12.50$.

(c) Matches (i); $y = 0.51x + 30$.

(d) Matches (iv); $y = -100x + 600$.

Chapter 1 Quiz Yourself

1. (a)



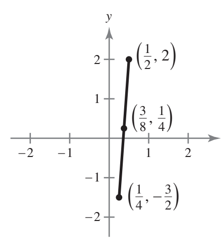
(b) $d = \sqrt{(-3-3)^2 + (1-(-2))^2}$

$= \sqrt{36 + 9}$

$= 3\sqrt{5}$

(c) Midpoint $= \left(\frac{-3+3}{2}, \frac{1-2}{2}\right) = \left(0, -\frac{1}{2}\right)$

2. (a)



(b) $d = \sqrt{\left(\frac{1}{2} - \frac{1}{4}\right)^2 + \left(2 - \left(-\frac{3}{2}\right)\right)^2}$

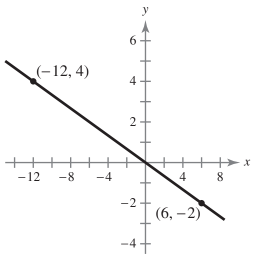
$= \sqrt{\frac{1}{16} + \frac{49}{4}}$

$= \sqrt{\frac{197}{16}}$

$= \frac{1}{4}\sqrt{197}$

(c) Midpoint $= \left(\frac{\frac{1}{2} + \frac{1}{4}}{2}, \frac{2 - \frac{3}{2}}{2}\right) = \left(\frac{3}{8}, \frac{1}{4}\right)$

3. (a)



(b) $d = \sqrt{(6-(-12))^2 + (-2-4)^2}$

$= \sqrt{18^2 + (-6)^2}$

$= \sqrt{324 + 36}$

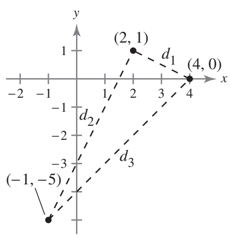
$= \sqrt{360}$

$= 6\sqrt{10}$

≈ 18.97

(c) Midpoint $= \left(\dfrac{-12+6}{2}, \dfrac{4+(-2)}{2}\right) = (-3, 1)$

4.



$a = \sqrt{(2-4)^2 + (1-0)^2} = \sqrt{5}$

$b = \sqrt{(2-(-1))^2 + (1-(-5))^2} = 3\sqrt{5}$

$c = \sqrt{(-1-4)^2 + (-5-0)^2} = 5\sqrt{2}$

$a^2 + b^2 = (\sqrt{5})^2 + (3\sqrt{5})^2 = (5\sqrt{2})^2 = c^2$

5. $(2011, 9810)$ and $(2013, 9992)$

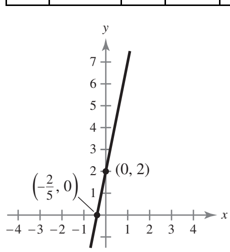
Midpoint $= \left(\dfrac{2011+2013}{2}, \dfrac{9810+9992}{2}\right)$

$= (2012, 9901)$

The population in 2012 was approximately 9901 thousand or 9,901,000.

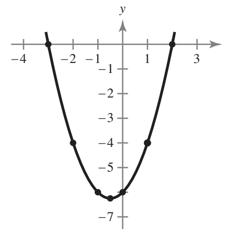
6. $y = 5x + 2$

x	$-\frac{2}{5}$	0	$\frac{1}{5}$	1
y	0	2	3	7



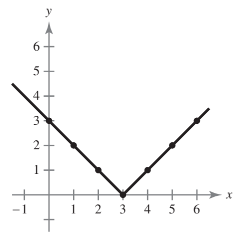
7. $y = x^2 + x - 6$

x	−3	−2	−1	−0.5	0	1	2
y	0	−4	−6	−6.25	−6	−4	0



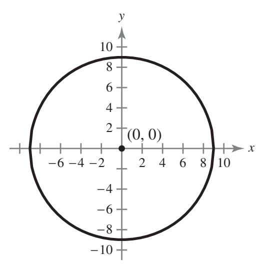
8. $y = |x - 3|$

x	0	1	2	3	4	5	6
y	3	2	1	0	1	2	3



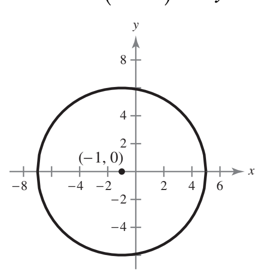
9. $(x-0)^2 + (y-0)^2 = 9^2$

$x^2 + y^2 = 81$



10. $(x-(-1))^2 + (y-0)^2 = 6^2$

$(x+1)^2 + y^2 = 36$



11. The radius is the distance between $(2, -2)$ and $(-1, 2)$.

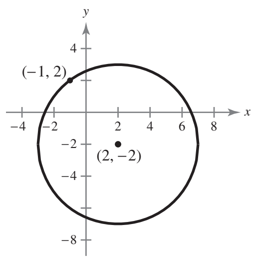
$$r = \sqrt{(-1-2)^2 + (2-(-2)^2)}$$

$$r = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Using the center $(2, -2)$ and radius $r = 5$:

$$(x-2)^2 + (y-(-2))^2 = 5^2$$

$$(x-2)^2 + (y+2)^2 = 25$$



12. $C = 4.55x + 12{,}500$

$R = 7.19x$

$R = C$

$7.19x = 4.55x + 12{,}500$

$2.64x = 12{,}500$

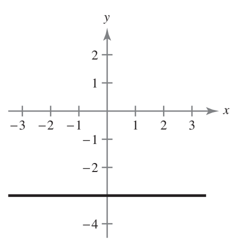
$x \approx 4734.8$

The company must sell 4735 units to break even.

13. $y = mx + b$

$y = 0x - 3$

$y = -3$

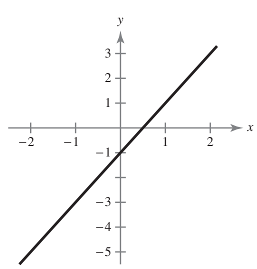


14. $y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 1)$

$y - 1 = 2x - 2$

$y = 2x - 1$

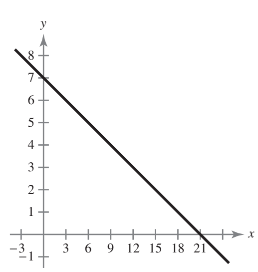


15. $y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{3}(x - 6)$

$y - 5 = -\frac{1}{3}x + 2$

$y = -\frac{1}{3}x + 7$

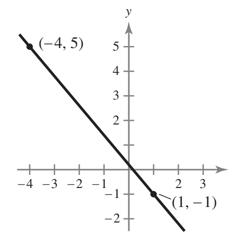


16. $(1, -1), (-4, 5)$

$$m = \frac{5+1}{-4-1} = -\frac{6}{5}$$

$$y + 1 = -\frac{6}{5}(x - 1)$$

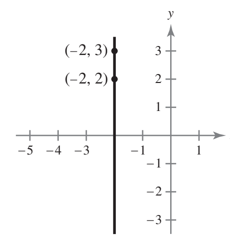
$$y = -\frac{6}{5}x + \frac{1}{5}$$



17. $(-2, 3), (-2, 2)$

$$m = \frac{2-3}{-2+2} = \text{undefined}$$

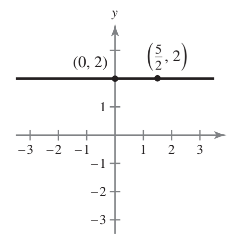
Because the slope is undefined, the line is vertical and its equation is $x = -2$.



18. $\left(\frac{5}{2}, 2\right), (0, 2)$

$$m = \frac{2-2}{0-\frac{5}{2}} = 0$$

Because the slope is 0, the line is horizontal and its equation is $y = 2$.



19. Given line: $y = -\frac{1}{4}x - \frac{1}{2}$, $m = -\frac{1}{4}$

(a) Parallel: $m_1 = -\frac{1}{4}$

$y + 5 = -\frac{1}{4}(x - 3)$

$y = -\frac{1}{4}x - \frac{17}{4}$

(b) Perpendicular: $m_2 = 4$

$y + 5 = 4(x - 3)$

$y = 4x - 17$

20. Let $t = 11$ correspond to 2011.

$(11, 1{,}330{,}000), (15, 1{,}800{,}000)$

$$m = \frac{1{,}800{,}000 - 1{,}330{,}000}{15 - 11} = \frac{470{,}000}{4} = 117{,}500$$

$y - 1{,}330{,}000 = 117{,}500(x - 11)$

$y - 1{,}330{,}000 = 117{,}500x - 1{,}292{,}500$

$y = 117{,}500x + 37{,}500$

For 2019, let $t = 19$.

$y = 117{,}500(19) + 37{,}500 = \$2{,}270{,}000$

For 2022, let $t = 22$.

$y = 117{,}500(22) + 37{,}500 = \$2{,}622{,}500$

21. The daily cost C equals the cost for lodging and meals plus the cost per mile driven, x.

$C = 218 + 0.56x$

22. (a) Let $t = 9$ correspond to 2013 and S equal salary.

2013: $(13, 35{,}700)$ and 2015: $(15, 39{,}100)$

$$m = \frac{39{,}100 - 35{,}700}{15 - 13} = \frac{3400}{2} = 1700$$

$$S - S_1 = m(t - t_1)$$

$$S - 35{,}700 = 1700(t - 13)$$

$$S - 35{,}700 = 1700t - 22{,}100$$

$$S = 1700t + 13{,}600$$

(b) For 2020, let $t = 20$.

$$S = 1700(20) + 13{,}600 = \$47{,}600$$

Section 1.4 Functions

Skills Warm Up

1. $5(-1)^2 - 6(-1) + 9 = 5(1) + 6 + 9 = 20$

2. $(-2)^3 + 4(-2)^2 - 12 = -8 + 4(4) - 12 = -8 + 16 - 12 = -4$

3. $(x - 2)^2 + 5x - 10 = x^2 - 4x + 4 + 5x - 10 = x^2 + x - 6$

4.
$$\begin{aligned}(3 - x) + (x + 3)^3 &= (3 - x) + (x + 3)(x^2 + 6x + 9)\\ &= (3 - x) + x^3 + 3x^2 + 6x^2 + 18x + 9x + 27\\ &= x^3 + 9x^2 + 26x + 30\end{aligned}$$

5. $\dfrac{1}{1 - (1 - x)} = \dfrac{1}{1 - 1 + x} = \dfrac{1}{x}$

6. $3 + \dfrac{2x - 7}{x} = \dfrac{3x}{x} + \dfrac{2x - 7}{x} = \dfrac{3x + 2x - 7}{x} = \dfrac{5x - 7}{x}$

7.
$$\begin{aligned}2x + y - 6 &= 11\\ y &= -2x + 17\end{aligned}$$

8.
$$\begin{aligned}5y - 6x^2 - 1 &= 0\\ 5y &= 6x^2 + 1\\ y &= \frac{6x^2 + 1}{5}\\ &= \frac{6}{5}x^2 + \frac{1}{5}\end{aligned}$$

9.
$$\begin{aligned}(y - 3)^2 &= 5 + (x + 1)^2\\ y - 3 &= \sqrt{5 + (x + 1)^2}\\ y - 3 &= \sqrt{5 + x^2 + 2x + 1}\\ y &= \sqrt{x^2 + 2x + 6} + 3\end{aligned}$$

10.
$$\begin{aligned}y^2 - 4x^2 &= 2\\ y^2 &= 2 + 4x^2\\ y &= \sqrt{2 + 4x^2}\end{aligned}$$

11.
$$\begin{aligned}x &= \frac{2y - 1}{4}\\ 4x &= 2y - 1\\ 4x + 1 &= 2y\\ \frac{4x + 1}{2} &= y\\ 2x + \frac{1}{2} &= y\end{aligned}$$

12.
$$\begin{aligned}x &= \sqrt[3]{2y - 1}\\ x^3 &= 2y - 1\\ -2y &= -x^3 - 1\\ y &= \tfrac{1}{2}x^3 + \tfrac{1}{2}\end{aligned}$$

1. $x^2 + y^2 = 16$

$$y^2 = 16 - x^2$$

$$y = \pm\sqrt{16 - x^2}$$

y is *not* a function of x since there are two values of y for some x.

2. $y = \pm\sqrt{4 - x}$

y is *not* a function of x since there are two values of y for some x.

3. $\frac{1}{2}x - 6y = -3$

$$y = \frac{1}{12}x + \frac{1}{2}$$

y *is* a function of x since there is only one value of y for each x.

4. $y = \dfrac{3x + 5}{2}$

y *is* a function of x since there is only one value of y for each x.

5. $y = 4 - x^2$

y *is* a function of x since there is only one value of y for each x.

6. $x^2 + y^2 + 2x = 0$

$$y^2 = -x^2 - 2x$$

$$y = \pm\sqrt{-x^2 - 2x}$$

y is *not* a function of x since there are two values of y for some x.

7. $y = |x + 2|$

y *is* a function of x since there is only one value of y for each x.

8. $x^2y^2 - 3x^2 + 4y^2 = 0$

$$x^2y^2 + 4y^2 = 3x^2$$

$$y^2(x^2 + 4) = 3x^2$$

$$y^2 = \frac{3x^2}{x^2 + 4}$$

$$y = \pm\sqrt{\frac{3x^2}{x^2 + 4}}$$

y is *not* a function of x since are two values of y for some x.

9. y is *not* a function of x.

10. y *is* a function of x.

11. y *is* a function of x.

12. y is *not* a function of x.

13. Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

14. Domain: $\left[\frac{3}{2}, \infty\right)$

Range: $[0, \infty)$

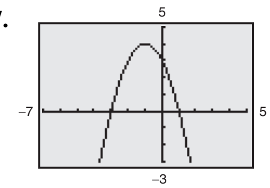
15. Domain: $[-2, 2]$

Range: $[0, 2]$

16. Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

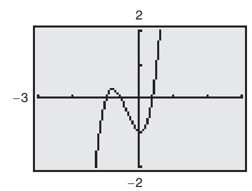
17.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 4]$

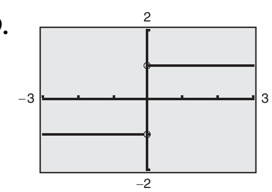
18.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

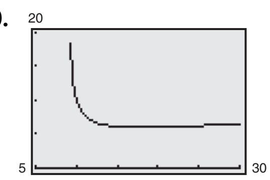
19.



Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $\{-1, 1\}$

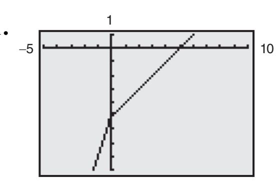
20.



Domain: $(9, \infty)$

Range: $[18, \infty)$

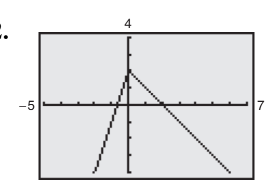
21.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

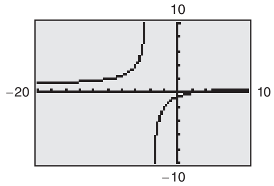
22.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

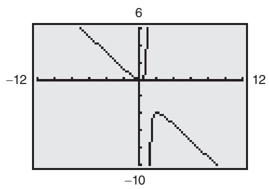
23.



Domain: $(-\infty, -4) \cup (-4, \infty)$

Range: $(-\infty, 1) \cup (1, \infty)$

24.



Domain: $(-\infty, 1) \cup (1, \infty)$

Range: $(-\infty, -4] \cup [0, \infty)$

25. $f(x) = 3x - 2$

(a) $f(0) = 3(0) - 2 = -2$

(b) $f(5) = 3(5) - 2 = 13$

(c) $f(x - 1) = 3(x - 1) - 2 = 3x - 3 - 2 = 3x - 5$

26. $f(x) = x^2 - 4x + 1$

(a) $f(-1) = (-1)^2 - 4(-1) + 1 = 6$

(b) $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = -\frac{3}{4}$

(c) $f(c + 2) = (c + 2)^2 - 4(c + 2) + 1$

$= c^2 + 4c + 4 - 4c - 8 + 1$

$= c^2 - 3$

27. $g(x) = \dfrac{1}{x}$

(a) $g\left(\dfrac{1}{5}\right) = \dfrac{1}{\frac{1}{5}} = 5$

(b) $g(-0.6) = \dfrac{1}{-0.6} = -\dfrac{5}{3}$

(c) $g(x + 4) = \dfrac{1}{x + 4}$

28. $f(x) = |x| + 4$

(a) $f(-3) = |-3| + 4 = 7$

(b) $f(0.8) = |0.8| + 4 = 4.8$

(c) $f(x + 2) = |x + 2| + 4$

29. $\dfrac{f(x + \Delta x) - f(x)}{\Delta x}$

$$= \frac{(x + \Delta x)^2 - 5(x + \Delta x) + 2 - (x^2 - 5x + 2)}{\Delta x}$$

$$= \frac{[x^2 + 2x\Delta x + (\Delta x)^2 - 5x + 5\Delta x + 2] - [x^2 - 5x + 2]}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 + 5\Delta x}{\Delta x}$$

$$= 2x + \Delta x + 5, \quad \Delta x \neq 0$$

30. $\dfrac{h(x + \Delta x) - h(x)}{\Delta x}$

$$= \frac{(x + \Delta x)^2 + (x + \Delta x) + 3 - (x^2 + x + 3)}{\Delta x}$$

$$= \frac{\left[x^2 + 2x\Delta x + (\Delta x)^2 + x + \Delta x + 3\right] - \left[x^2 + x + 3\right]}{\Delta x}$$

$$= \frac{2x\Delta x + (\Delta x)^2 + \Delta x}{\Delta x}$$

$$= \frac{\Delta x(2x + \Delta x + 1)}{\Delta x}$$

$$= 2x + \Delta x + 1, \ \Delta x \neq 0$$

31. $\dfrac{g(4 + \Delta x) - g(4)}{\Delta x}$

$$= \frac{\sqrt{4 + \Delta x + 1} - \sqrt{4 + 1}}{\Delta x}$$

$$= \frac{\sqrt{\Delta x + 5} - \sqrt{5}}{\Delta x} \cdot \frac{\sqrt{\Delta x + 5} + \sqrt{5}}{\sqrt{\Delta x + 5} + \sqrt{5}}$$

$$= \frac{(\Delta x + 5) - 5}{\Delta x\left[\sqrt{\Delta x + 5} + \sqrt{5}\right]}$$

$$= \frac{\Delta x}{\Delta x\left[\sqrt{\Delta x + 5} + \sqrt{5}\right]}$$

$$= \frac{1}{\sqrt{\Delta x + 5} + \sqrt{5}}, \ \Delta x \neq 0$$

32. $\dfrac{f(x) - f(2)}{x - 2} = \dfrac{\dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{2}}}{x - 2}$

$$= \frac{\dfrac{1}{\sqrt{x}} \cdot \dfrac{2\sqrt{x}}{2\sqrt{x}} - \dfrac{1}{\sqrt{2}} \cdot \dfrac{x\sqrt{2}}{x\sqrt{2}}}{x - 2}$$

$$= \frac{\dfrac{2\sqrt{x} - x\sqrt{2}}{2x}}{x - 2}$$

$$= \frac{2\sqrt{x} - x\sqrt{2}}{2x(x - 2)}$$

33. $\dfrac{f(x+\Delta x)-f(x)}{\Delta x} = \dfrac{\dfrac{1}{x+\Delta x-2}-\dfrac{1}{x-2}}{\Delta x}$

$= \dfrac{(x-2)-(x+\Delta x-2)}{(x+\Delta x-2)(x-2)\Delta x}$

$= \dfrac{-1}{(x+\Delta x-2)(x-2)},\ \Delta x \neq 0$

34. $\dfrac{f(x+\Delta x)-f(x)}{\Delta x} = \dfrac{\dfrac{1}{x+\Delta x+4}-\dfrac{1}{x+4}}{\Delta x}$

$= \dfrac{(x+4)-(x+\Delta x+4)}{\Delta x[x+\Delta x+4][x+4]}$

$= \dfrac{-1}{(x+\Delta x+4)(x+4)},\ \Delta x \neq 0$

35. (a) $f(x)+g(x) = (2x-5)+(4-3x) = -x-1$

(b) $f(x)-g(x) = (2x-5)-(4-3x) = 5x-9$

(c) $f(x)\cdot g(x) = (2x-5)(4-3x) = 8x-6x^2-20+15x = -6x^2+23x-20$

(d) $f(x)/g(x) = \dfrac{2x-5}{4-3x},\ x \neq \dfrac{3}{4}$

(e) $f(g(x)) = f(4-3x) = 2(4-3x)-5 = 8-6x-5 = -6x+3$

(f) $g(f(x)) = g(2x-5) = 4-3(2x-5) = 4-6x+15 = -6x+19$

36. (a) $f(x)+g(x) = x^2+5+\sqrt{1-x},\ x \le 1$

(b) $f(x)-g(x) = x^2+5-\sqrt{1-x},\ x \le 1$

(c) $f(x)\cdot g(x) = \left(\sqrt{1-x}\right)(x^2+5),\ x \le 1$

(d) $f(x)/g(x) = \dfrac{x^2+5}{\sqrt{1-x}},\ x \le 1$

(e) $f(g(x)) = 6-x,\ x \le 1$

(f) $g(f(x))$ is undefined

37. (a) $f(x)+g(x) = x^2+1+x-1 = x^2+x$

(b) $f(x)-g(x) = x^2+1-x+1 = x^2-x+2$

(c) $f(x)\cdot g(x) = (x^2+1)(x-1) = x^3-x^2+x-1$

(d) $f(x)/g(x) = \dfrac{x^2+1}{x-1},\ x \neq 1$

(e) $f(g(x)) = (x-1)^2+1 = x^2-2x+2$

(f) $g(f(x)) = x^2+1-1 = x^2$

38. (a) $f(x)+g(x) = \dfrac{x^4+x^3+x}{x+1},\ x \neq -1$

(b) $f(x)-g(x) = \dfrac{x-x^3-x^4}{x+1},\ x \neq -1$

(c) $f(x)\cdot g(x) = \dfrac{x^4}{x+1},\ x \neq -1$

(d) $f(x)/g(x) = \dfrac{x}{x^4+x^3},\ x \neq 0, -1$

(e) $f(g(x)) = \dfrac{x^3}{x^3+1},\ x \neq -1$

(f) $g(f(x)) = \dfrac{x^3}{(x+1)^3},\ x \neq -1$

39. (a) $f(g(1)) = f(1^2-1) = f(0) = \sqrt{0} = 0$

(b) $g(f(1)) = g\left(\sqrt{1}\right) = g(1) = 1^2-1 = 0$

(c) $g\left(f\left(\tfrac{1}{2}\right)\right) = g\left(\sqrt{\tfrac{1}{2}}\right) = \left(\sqrt{\tfrac{1}{2}}\right)^2-1 = \tfrac{1}{2}-1 = -\tfrac{1}{2}$

(d) $f\left(g\left(-\sqrt{5}\right)\right) = f(4) = \sqrt{4} = 2$

(e) $f(g(x)) = f(x^2-1) = \sqrt{x^2-1}$

(f) $g(f(x)) = g\left(\sqrt{x}\right) = x-1,\ x \ge 0$

40. (a) $f(g(2)) = f(3) = \dfrac{1}{3}$

(b) $g(f(2)) = g\left(\dfrac{1}{2}\right) = -\dfrac{3}{4}$

(c) $f(g(-3)) = f(8) = \dfrac{1}{8}$

(d) $g\left(f\left(\dfrac{1}{\sqrt{2}}\right)\right) = g\left(\sqrt{2}\right) = 1$

(e) $f(g(x)) = \dfrac{1}{x^2-1},\ x \neq \pm 1$

(f) $g(f(x)) = \dfrac{1}{x^2}-1,\ x \neq 0$

41. $f(x) = 4x$

$f^{-1}(x) = \tfrac{1}{4}x$

$f\left(f^{-1}(x)\right) = 4\left(\tfrac{1}{4}x\right) = x$

$f^{-1}(f(x)) = \tfrac{1}{4}(4x) = x$

42. $f(x) = \frac{1}{3}x$

$f^{-1}(x) = 3x$

$f(f^{-1}(x)) = \frac{1}{3}(3x) = x$

$f^{-1}(f(x)) = 3\left(\frac{1}{3}x\right) = x$

43. $f(x) = x + 12$

$f^{-1}(x) = x - 12$

$f(f^{-1}(x)) = (x - 12) + 12 = x$

$f^{-1}(f(x)) = (x + 12) - 12 = x$

44. $f(x) = x - 3$

$f^{-1}(x) = x + 3$

$f(f^{-1}(x)) = (x + 3) - 3 = x$

$f^{-1}(f(x)) = (x - 3) + 3 = x$

45. $f(x) = 2x - 3 = y$

$2y - 3 = x$

$2y = x + 3$

$y = \dfrac{x + 3}{2}$

$f'(x) = \dfrac{1}{2}x + \dfrac{3}{2}$

46. $f(x) = 5 - \frac{3}{4}x = y$

$5 - \frac{3}{4}y = x$

$-\frac{3}{4}y = x - 5$

$-3y = 4x - 20$

$y = -\frac{3}{4}x + \frac{20}{3}$

$f'(x) = -\frac{4}{3}x + \frac{20}{3}$

47. $f(x) = \frac{3}{2}x + 1 = y$

$\frac{3}{2}y + 1 = x$

$3y + 2 = 2x$

$3y = 2x - 2$

$y = \frac{2}{3}x - \frac{2}{3}$

$f^{-1}(x) = \frac{2}{3}x - \frac{2}{3}$

48. $f(x) = -6x - 4 = y$

$-6y - 4 = x$

$-6y = x + 4$

$y = -\frac{1}{6}x - \frac{2}{3}$

$f^{-1}(x) = -\frac{1}{6}x - \frac{2}{3}$

49. $f(x) = x^5 = y$

$y^5 = x$

$y = \sqrt[5]{x}$

$f^{-1}(x) = \sqrt[5]{x}$

50. $f(x) = x^3 = y$

$y^3 = x$

$y = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x}$

51. $f(x) = \dfrac{1}{x} = y$

$\dfrac{1}{y} = x$

$y = \dfrac{1}{x}$

$f^{-1}(x) = \dfrac{1}{x}$

52. $f(x) = -\dfrac{2}{x} = y$

$-\dfrac{2}{y} = x$

$-\dfrac{1}{y} = \dfrac{1}{2}x$

$y = -\dfrac{2}{x}$

$f^{-1}(x) = -\dfrac{2}{x}$

53. $f(x) = \sqrt{9 - x^2} = y,\ 0 \le x \le 3$

$\sqrt{9 - y^2} = x$

$9 - y^2 = x^2$

$y^2 = 9 - x^2$

$y = \sqrt{9 - x^2}$

$f^{-1}(x) = \sqrt{9 - x^2},\ \ 0 \le x \le 3$

54. $f(x) = \sqrt{x^2 - 4} = y,\ x \ge 2$

$\sqrt{y^2 - 4} = x$

$y^2 = x^2 + 4$

$y = \sqrt{x^2 + 4},\ x \ge 0$

$f^{-1}(x) = \sqrt{x^2 + 4},\ x \ge 0$

55. $f(x) = x^{2/3} = y,\ x \geq 0$

$y^{2/3} = x$

$y = x^{3/2}$

$f^{-1}(x) = x^{3/2}$

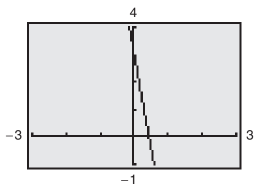
56. $f(x) = x^{3/5} = y$

$y^{3/5} = x$

$y = x^{5/3}$

$f^{-1}(x) = x^{5/3}$

57.



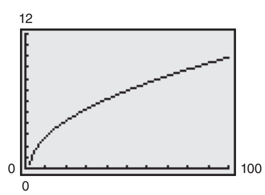
$f(x) = 3 - 7x$ is one-to-one.

$y = 3 - 7x$

$x = 3 - 7y$

$y = \dfrac{3 - x}{7}$

58.



$f(x) = \sqrt{x - 2}$ is one-to-one.

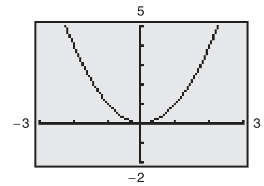
$y = \sqrt{x - 2}$

$x = \sqrt{y - 2}$

$x^2 = y - 2$

$y = x^2 + 2,\ x \geq 0$

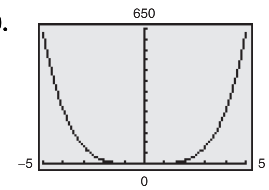
59.



$f(x) = x^2$

f is *not* one-to-one because $f(1) = 1 = f(-1)$.

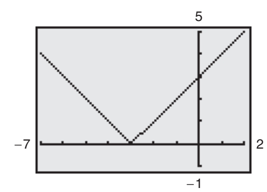
60.



$f(x) = x^4$ is *not* one-to-one because

$f(2) = 16 = f(-2)$.

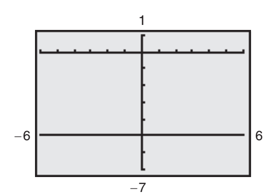
61. $f(x) = |x + 3|$



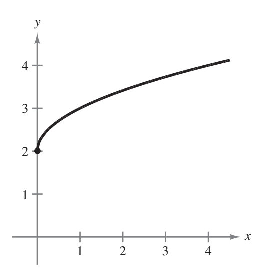
f is *not* one-to-one because $f(-5) = 2 = f(-1)$.

62. $f(x) = -5$ is *not* one-to-one because

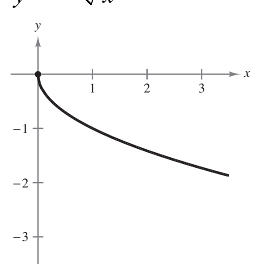
$f(1) = -5 = f(-1)$.



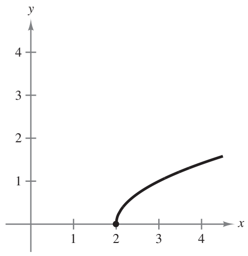
63. (a) $y = \sqrt{x} + 2$



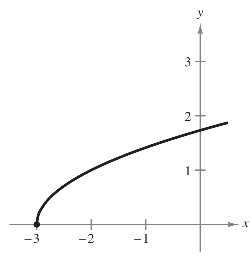
(b) $y = -\sqrt{x}$



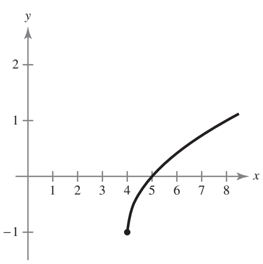
(c) $y = \sqrt{x - 2}$



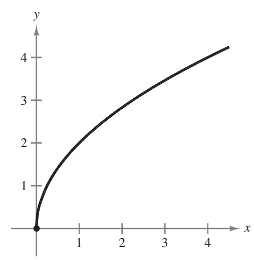
(d) $y = \sqrt{x + 3}$



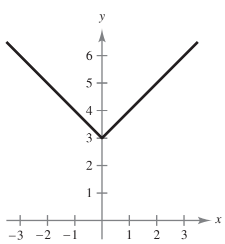
(e) $y = \sqrt{x - 4} - 1$



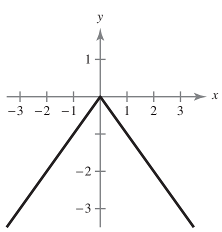
(f) $y = 2\sqrt{x}$



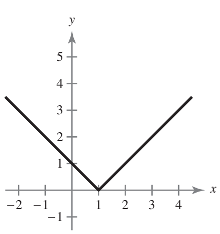
64. (a)



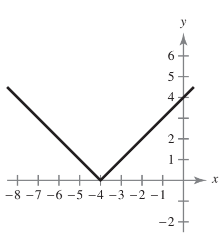
(b)



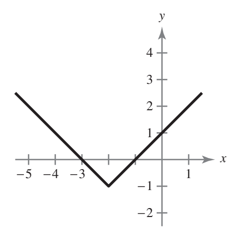
(c)



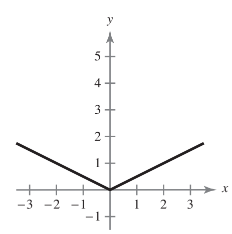
(d)



(e)



(f)



65. (a) Shifted three units to the left: $y = (x + 3)^2$

(b) Shifted six units to the left, three units downward, and reflected: $y = -(x + 6)^2 - 3$

66. (a) Stretched by a factor of $\frac{1}{8}$: $y = \frac{1}{8}x^3$

(b) Stretched by a factor of 2, and reflected: $y = -2x^3$

67. (a) 2004: \$190 billion
2006: \$225 billion
2013: \$270 billion

(b) 2004: $d(4) = 15.73(4) + 128.3 = \$191.22$ billion

2006: $d(6) = -0.620(6)^2 + 18.11(6) + 138.8 = \$225.14$ billion

2013: $d(13) = -0.620(13)^2 + 18.11(13) + 138.8 = \$269.45$ billion

The model fits the data well.

68. (a) T is a function of t because for any value of t (time of day), there is exactly one value of T (temperature in the house).

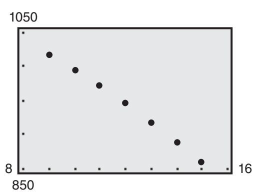
(b) $T(4) \approx 60°\text{F}$

$T(15) \approx 72°\text{F}$

(c) H is a horizontal shift of T, 1 hour to the right or 1 hour later.

(d) H is a vertical shift of T, 1 degree upward or 1 degree increase in setting.

69. $$\begin{aligned} R_{\text{TOTAL}} &= R_1 + R_2 \\ &= 690 - 8t - 0.8t^2 + 458 + 0.78t \\ &= -0.8t^2 - 7.22t + 1148,\ t = 9, 10, \ldots, 15 \end{aligned}$$



70. $$\begin{aligned} B(t) - D(t) &= 4.917t^3 - 124.71t^2 - 925.9t + 2308 - \left(-7.083t^3 + 222.64t^2 - 281.8t + 10{,}104\right) \\ &= 12.0t^3 - 347.35t^2 + 3207.7t - 7796 \end{aligned}$$

$B(t) - D(t)$ is the function that yields the increase and/or decrease in people living in the United States from 2008 to 2012.

71. (a) $C = 2.89x + 8000$

(b) $\overline{C} = \dfrac{C}{x} = \dfrac{2.89x + 8000}{x} = 2.89 + \dfrac{8000}{x}$

(c) $$\begin{aligned} 2.89 + \frac{8000}{x} &< 6.89 \\ \frac{8000}{x} &< 4 \\ \frac{8000}{4} &< x \text{ because } x > 0. \\ 2000 &< x \end{aligned}$$

Must sell 2000 units before the average cost per unit falls below the selling price.

72. (a) $1 + 0.01x = \dfrac{14.75}{p}$

$$x = \frac{(14.75/p) - 1}{0.01} = \frac{14.75 - p}{0.01p} = \frac{100(14.75 - p)}{p} = \frac{1475}{p} - 100$$

(b) $x = \dfrac{100(14.75 - 10)}{10} = 47.5 \approx 48$ units

73. (a) $C(x) = 70x + 500$

$x(t) = 40t$

$C(x(t)) = 70(40t) + 500$

$= 2800t + 500$

C is the weekly cost per t hours of production.

(b) $C(x(4)) = 2800(4) + 500 = \$11{,}700$

(c) $C(x(t)) = 18{,}000$

$2800t + 500 = 18{,}000$

$2800t = 17{,}500$

$t = \dfrac{17{,}500}{2800} = 6.25$ hr

74. (a) Revenue $= R = rn = [15 - 0.05(n - 80)]n = 19n - 0.05n^2$

(b)

n	100	125	150	175	200	225	250
R	1400	1593.75	1725	1793.75	1800	1743.75	1625

(c) The revenue increases and then decreases as n gets larger, so it is not a good formula for the bus company to use.

75. (a) Cost $= C = 98{,}000 + 12.30x$

(b) Revenue $= R = 17.98x$

(c) Profit $= R - C = 17.98x - (12.30x + 98{,}000) = 5.68x - 98{,}000$

76. (a) If $0 \le x \le 100$, then $p = 90$. If $100 < x \le 1600$, then $p = 90 - 0.01(x - 100) = 91 - 0.01x$. If $x > 1600$, then $p = 75$. Thus,

$$p = \begin{cases} 90, & 0 \le x \le 100 \\ 91 - 0.01x, & 100 < x \le 1600. \\ 75, & x > 1600 \end{cases}$$

(b) $P = px - 60x$

$$P = \begin{cases} 90x - 60x, & 0 \le x \le 100 \\ (91 - 0.01x)x - 60x, & 100 < x \le 1600 \\ 75x - 60x, & x > 1600 \end{cases}$$

$$= \begin{cases} 30x, & 0 \le x \le 100 \\ 31x - 0.01x^2, & 100 < x \le 1600 \\ 15x, & x > 1600 \end{cases}$$

Answers for Exercises 77–84 are not unique. Sample answers are given.

77. $f(x) = (x - 1)^2,\ x \ge 1$

$y = (x - 1)^2$

$x = (y - 1)^2$

$\pm\sqrt{x} = y - 1$

$1 \pm \sqrt{x} = y$

$1 + \sqrt{x} = f^{-1}(x)$

Domain of f: $[1, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[1, \infty)$

78.
$$\begin{aligned} f(x) &= (x+2)^2,\ x \ge -2 \\ y &= (x+2)^2 \\ x &= (x+2)^2 \\ \pm\sqrt{x} &= x+2 \\ -2 \pm \sqrt{x} &= y \\ -2 + \sqrt{x} &= f^{-1}(x) \end{aligned}$$

Domain of f: $[-2, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-2, \infty)$

79.
$$\begin{aligned} f(x) &= |x+4|,\ x \ge -4 \\ y &= |x+4| \\ x &= |y+4| \\ \pm x &= y+4 \\ -4 \pm x &= y \\ x - 4 &= f^{-1}(x) \end{aligned}$$

Domain of f: $[-4, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[-4, \infty)$

80.
$$\begin{aligned} f(x) &= |x-3|,\ x \ge 3 \\ y &= |x-3| \\ x &= |y-3| \\ \pm x &= y-3 \\ 3 \pm x &= y \\ x + 3 &= f^{-1}(x) \end{aligned}$$

Domain of f: $[3, \infty)$

Range of f: $[0, \infty)$

Domain of f^{-1}: $[0, \infty)$

Range of f^{-1}: $[3, \infty)$

81.
$$\begin{aligned} f(x) &= -2x^2 + 1,\ x \ge 0 \\ y &= -2x^2 + 1 \\ x &= -2y^2 + 1 \\ x - 1 &= -2y^2 \\ \frac{x-1}{-2} &= y^2 \\ \frac{1-x}{2} &= y^2 \\ \pm\sqrt{\frac{1-x}{2}} &= y \\ \sqrt{\frac{1-x}{2}} &= f^{-1}(x) \\ \frac{\sqrt{2(1-x)}}{2} &= f^{-1}(x) \end{aligned}$$

Domain of f: $[0, \infty)$

Range of f: $(-\infty, 1]$

Domain of f^{-1}: $(-\infty, 1]$

Range of f^{-1}: $[0, \infty)$

82.
$$\begin{aligned} f(x) &= \tfrac{1}{2}x^2 - 4,\ x \ge 0 \\ y &= \tfrac{1}{2}x^2 - 4 \\ x &= \tfrac{1}{2}y^2 - 4 \\ x + 4 &= \tfrac{1}{2}y^2 \\ 2(x+4) &= y^2 \\ \pm\sqrt{2(x+4)} &= y \\ \sqrt{2(x+4)} &= f^{-1}(x) \end{aligned}$$

Domain of f: $[0, \infty)$

Range of f: $(-4, \infty]$

Domain of f^{-1}: $(-4, \infty]$

Range of f^{-1}: $[0, \infty)$

83.
$$\begin{aligned} f(x) &= |x+1| - 2,\ x \ge -1 \\ y &= |x+1| - 2 \\ x &= |y+1| - 2 \\ x+2 &= |y+1| \\ \pm(x+2) &= y+1 \\ -1 \pm (x+2) &= y \\ -1 + (x+2) &= f^{-1}(x) \\ x+1 &= f^{-1}(x) \end{aligned}$$

Domain of f: $[-1, \infty)$

Range of f: $[-2, \infty)$

Domain of f^{-1}: $[-2, \infty)$

Range of f^{-1}: $[-1, \infty)$

84.
$$\begin{aligned} f(x) &= -|x-2| + 3,\ x \ge 2 \\ y &= -|x-2| + 3 \\ x &= -|y-2| + 3 \\ x-3 &= -|y-2| \\ -(x-3) &= |y-2| \\ \mp(x-3) &= y-2 \\ 2 \mp (x-3) &= f^{-1}(x) \\ 5 - x &= f^{-1}(x) \end{aligned}$$

Domain of f: $[2, \infty)$

Range of f: $(-\infty, 3]$

Domain of f^{-1}: $(-\infty, 3]$

Range of f^{-1}: $[2, \infty)$

85. Answers will vary.

Section 1.5 Limits

Skills Warm Up

1. $\dfrac{2x^3 + x^2}{6x} = \dfrac{x^2(2x+1)}{6x} = \dfrac{x(2x+1)}{6} = \dfrac{1}{6}x(2x+1)$

2. $\dfrac{x^5 + 9x^4}{x^2} = \dfrac{x^4(x+9)}{x^2} = x^3(x+9)$

3. $\dfrac{x^2 - 3x - 28}{x-7} = \dfrac{(x-7)(x+4)}{x-7} = x+4$

4. $\dfrac{x^2 + 11x + 30}{x+5} = \dfrac{(x+6)(x+5)}{x+5} = x+6$

5. $f(x) = x^2 - 3x + 3$

(a) $f(-1) = (-1)^2 - 3(-1) + 3 = 1 + 3 + 3 = 7$

(b) $f(c) = c^2 - 3c + 3$

(c) $f(x+h) = (x+h)^2 - 3(x+h) + 3$
$= x^2 + 2xh + h^2 - 3x - 3h + 3$

6. $f(x) = \begin{cases} 2x-2, & x < 1 \\ 3x+1, & x \ge 1 \end{cases}$

(a) $f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) - 2 = -1 - 2 = -3$

(b) $f(1) = 3(1) + 1 = 3 + 1 = 4$

(c) $f(t^2+1) = 3(t^2+1) + 1$
$= 3t^2 + 3 + 1$
$= 3t^2 + 4$

7. $f(x) = x^2 - 2x + 2$

$$\begin{aligned} &\frac{f(1+h) - f(1)}{h} \\ &= \frac{(1+h)^2 - 2(1+h) + 2 - (1^2 - 2(1) + 2)}{h} \\ &= \frac{1 + 2h + h^2 - 2 - 2h + 2 - 1 + 2 - 2}{h} \\ &= \frac{h^2}{h} \\ &= h \end{aligned}$$

8. $f(x) = 4x$

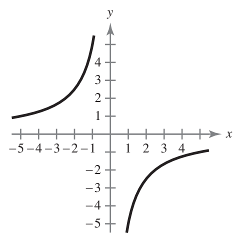
$$\begin{aligned} \frac{f(2+h) - f(2)}{h} &= \frac{4(2+h) - 4(2)}{h} \\ &= \frac{8 + 4h - 8}{h} \\ &= \frac{4h}{h} \\ &= 4 \end{aligned}$$

Skills Warm Up —*continued*—

9. $h(x) = -\dfrac{5}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

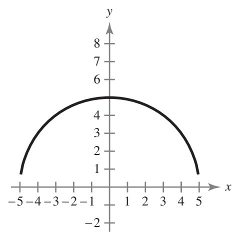
Range: $(-\infty, 0) \cup (0, \infty)$



10. $g(x) = \sqrt{25 - x^2}$

Domain: $[-5, 5]$

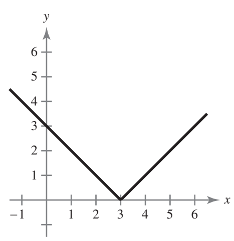
Range: $[0, 5]$



11. $f(x) = |x - 3|$

Domain: $(-\infty, \infty)$

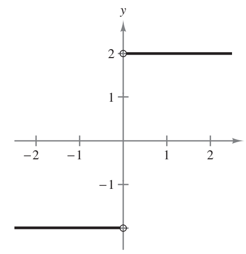
Range: $[0, \infty)$



12. $f(x) = \dfrac{2|x|}{x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $y = -2,\ y = 2$



13.

$$9x^2 + 4y^2 = 49$$
$$4y^2 = 49 - 9x^2$$
$$y^2 = \frac{49 - 9x^2}{4}$$
$$y = \frac{\pm\sqrt{49 - 9x^2}}{2}$$

Not a function of x (fails the vertical line test).

14.

$$2x^2y + 8x = 7y$$
$$2x^2y - 7y = -8x$$
$$y(2x^2 - 7) = -8x$$
$$y = -\frac{8x}{2x^2 - 7}$$

Yes, y is a function of x.

1. (a) $\lim_{x\to 2} f(x) = 4$

(b) $\lim_{x\to -1} f(x) = 1$

2. (a) $\lim_{x\to 1} f(x) = -2$

(b) $\lim_{x\to 3} f(x) = 0$

3. (a) $\lim_{x\to 0} g(x) = 1$

(b) $\lim_{x\to -1} g(x) = 3$

4. (a) $\lim_{x\to -2} h(x) = -5$

(b) $\lim_{x\to 0} h(x) = -3$

5.

x	5.9	5.99	5.999	6	6.001	6.01	6.1
$f(x)$	2.96	2.996	2.9996	?	3.0004	3.004	3.04

$\lim_{x\to 6} \dfrac{2x + 3}{5} = 3$

6.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	–1.79	–1.9799	–1.997999	?	–2.001999	–2.0199	–2.19

$\lim_{x\to 1} (x^2 - 4x + 1) = -2$

7.

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$	0.3448	0.3344	0.3334	?	0.3332	0.3322	0.3226

$\lim_{x\to 4} \dfrac{x - 4}{x^2 - 5x + 4} = \dfrac{1}{3}$

8.

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	0.2564	0.2506	0.2501	?	0.2499	0.2494	0.2439

$$\lim_{x \to 2} \frac{x - 2}{x^2 - 4} = \frac{1}{4}$$

9.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.1252	0.1250	0.1250	?	0.1250	0.1250	0.1248

$$\lim_{x \to 0} \frac{\sqrt{x + 16} - 4}{x} = 0.125 = \frac{1}{8}$$

10.

x	–0.1	–0.01	–0.001	0	0.001	0.01	0.1
$f(x)$	0.3581	0.3540	0.3536	?	0.3535	0.3531	0.3492

$$\lim_{x \to 0} \frac{\sqrt{x + 2} - \sqrt{2}}{x} = \frac{1}{2\sqrt{2}} \approx 0.3536$$

11.

x	–4.1	–4.01	–4.001	–4	–3.999	–3.99	–3.9
$f(x)$	2.5	25	250	?	–250	–25	–2.5

The limit does not exist.

12.

x	–2.1	–2.01	–2.001	–2	–1.999	–1.99	–1.9
$f(x)$	–2.5	–25	–250	?	250	25	2.5

The limit does not exist.

13. $\lim_{x \to 3} 6 = 6$

14. $\lim_{x \to 5} 4 = 4$

15. $\lim_{x \to -2} x = -2$

16. $\lim_{x \to 10} x = 10$

17. $\lim_{x \to 7} x^2 = (7)^2 = 49$

18. $\lim_{x \to 3} x^3 = (3)^3 = 27$

19. $\lim_{x \to 36} \sqrt{x} = \sqrt{36} = 6$

20. $\lim_{x \to -1} \sqrt[3]{x} = \sqrt[3]{-1} = -1$

21. (a) $\lim_{x \to c}[f(x) + g(x)] = \lim_{x \to c} f(x) + \lim_{x \to c} g(x)$
$= 3 + 9$
$= 12$

(b) $\lim_{x \to c}[f(x)g(x)] = \left[\lim_{x \to c} f(x)\right]\left[\lim_{x \to c} g(x)\right]$
$= 3 \cdot 9$
$= 27$

(c) $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{3}{9} = \frac{1}{3}$

22. (a) $\lim_{x\to c}\left[f(x) + g(x)\right] = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$
$$= \frac{3}{2} + \frac{1}{2}$$
$$= 2$$

(b) $\lim_{x\to c}\left[f(x)\cdot g(x)\right] = \left[\lim_{x\to c} f(x)\right]\left[\lim_{x\to c} g(x)\right]$
$$= \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{3}{4}$$

(c) $\lim_{x\to c}\frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{3/2}{1/2} = 3$

23. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{16} = 4$

(b) $\lim_{x\to c}\left[3f(x)\right] = 3(16) = 48$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 16^2 = 256$

24. (a) $\lim_{x\to c}\sqrt{f(x)} = \sqrt{9} = 3$

(b) $\lim_{x\to c}\left(3f(x)\right) = 3(9) = 27$

(c) $\lim_{x\to c}\left[f(x)\right]^2 = 9^2 = 81$

25. $\lim_{x\to -3}(2x + 5) = \lim_{x\to -3} 2x + \lim_{x\to -3} 5 = 2(-3) + 5 = -1$

26. $\lim_{x\to -4}(4x + 3) = \lim_{x\to -4} 4x + \lim_{x\to -4} 3 = 4(-4) + 3 = -13$

27. $\lim_{x\to 1}\left(1 - x^2\right) = \lim_{x\to 1} 1 - \lim_{x\to 1} x^2 = 1 - 1^2 = 0$

28. $\lim_{x\to 2}\left(-x^2 + x - 2\right) = -\lim_{x\to 2} x^2 + \lim_{x\to 2} x - \lim_{x\to 2} 2$
$$= -4 + 2 - 2 = -4$$

29. $\lim_{x\to 3}\sqrt{x + 6} = \sqrt{3 + 6} = 3$

30. $\lim_{x\to 5}\sqrt[3]{x - 5} = \sqrt[3]{5 - 5} = \sqrt[3]{0} = 0$

31. $\lim_{x\to -3}\frac{2}{x + 2} = \frac{2}{-3 + 2} = -2$

32. $\lim_{x\to -2}\frac{3x + 1}{2 - x} = \frac{3(-2) + 1}{2 - (-2)} = \frac{-5}{4} = -\frac{5}{4}$

33. $\lim_{x\to -2}\frac{x^2 - 1}{2x} = \frac{(-2)^2 - 1}{2(-2)} = \frac{3}{-4} = -\frac{3}{4}$

34. $\lim_{x\to -8}\frac{3x}{x + 2} = \frac{3(-8)}{(-8) + 2} = \frac{-24}{-6} = 4$

35. $\lim_{x\to 5}\frac{\sqrt{x + 11} + 6}{x} = \frac{\sqrt{5 + 11} + 6}{5}$
$$= \frac{\sqrt{16} + 6}{5}$$
$$= \frac{4 + 6}{5} = \frac{10}{5} = 2$$

36. $\lim_{x\to 12}\frac{\sqrt{x - 3} - 2}{x} = \frac{\sqrt{12 - 3} - 2}{12}$
$$= \frac{\sqrt{9} - 2}{12} = \frac{3 - 2}{12} = \frac{1}{12}$$

37. $\lim_{x\to -3}\frac{x^2 - 9}{x + 3} = \lim_{x\to -3}\frac{(x + 3)(x - 3)}{x + 3}$
$$= \lim_{x\to -3}(x - 3) = -6$$

38. $\lim_{x\to -1}\frac{2x^2 - x - 3}{x + 1} = \lim_{x\to -1}\frac{(x + 1)(2x - 3)}{x + 1}$
$$= \lim_{x\to -1}(2x - 3) = -5$$

39. $\lim_{x\to 2}\frac{x^2 + 3x - 10}{x^2 - 4} = \lim_{x\to 2}\frac{(x + 5)(x - 2)}{(x + 2)(x - 2)}$
$$= \lim_{x\to 2}\frac{x + 5}{x + 2} = \frac{2 + 5}{2 + 2} = \frac{7}{4}$$

40. $\lim_{t\to 1}\frac{t^2 + t - 2}{t^2 - 1} = \lim_{t\to 1}\frac{(t - 1)(t + 2)}{(t + 1)(t - 1)}$
$$= \lim_{t\to 1}\frac{t + 2}{t + 1} = \frac{3}{2}$$

41. $\lim_{x\to -2}\frac{x^3 + 8}{x + 2} = \lim_{x\to -2}\frac{(x + 2)\left(x^2 - 2x + 4\right)}{x + 2}$
$$= \lim_{x\to -2}\left(x^2 - 2x + 4\right) = 12$$

42. $\lim_{x\to -3}\frac{x^3 + 27}{x + 3} = \lim_{x\to -3}\frac{(x + 3)\left(x^2 - 3x + 9\right)}{x + 3}$
$$= \lim_{x\to -3}\left(x^2 - 3x + 9\right) = (-3)^2 - 3(-3) + 9 = 9$$

43. $\lim_{\Delta x\to 0}\frac{2(x+\Delta x)-2x}{\Delta x} = \lim_{\Delta x\to 0}\frac{2x+2\Delta x-2x}{\Delta x}$
$= \lim_{\Delta x\to 0} 2 = 2$

44. $\lim_{\Delta x\to 0}\frac{4(x+\Delta x)-5-(4x-5)}{\Delta x} = \lim_{\Delta x\to 0}\frac{4\Delta x}{\Delta x} = 4$

45. $\lim_{\Delta t\to 0}\frac{(t+\Delta t)^2-5(t+\Delta t)-(t^2-5t)}{\Delta t} = \lim_{\Delta t\to 0}\frac{t^2+2t(\Delta t)+(\Delta t)^2-5t-5(\Delta t)-t^2+5t}{\Delta t}$
$= \lim_{\Delta t\to 0}\frac{2t(\Delta t)+(\Delta t)^2-5(\Delta t)}{\Delta t}$
$= \lim_{\Delta t\to 0} 2t+(\Delta t)-5$
$= 2t-5$

46. $\lim_{\Delta t\to 0}\frac{(t+\Delta t)^2-4(t+\Delta t)+2-(t^2-4t+2)}{\Delta t} = \lim_{\Delta t\to 0}\frac{t^2+2t\Delta t+(\Delta t)^2-4t-4\Delta t-t^2+4t}{\Delta t}$
$= \lim_{\Delta t\to 0}\frac{2t\Delta t+(\Delta t)^2-4\Delta t}{\Delta t}$
$= \lim_{\Delta t\to 0}(2t+\Delta t-4)$
$= 2t-4$

47. $\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4} = \lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}\cdot\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$
$= \lim_{x\to 4}\frac{(x+5)-9}{(x-4)\sqrt{x+5}+3}$
$= \lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x+5}+3)}$
$= \lim_{x\to 4}\frac{1}{\sqrt{x+5}+3} = \frac{1}{6}$

48. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}\cdot\lim_{x\to 3}\frac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$
$= \lim_{x\to 3}\frac{(x+1)-4}{(x-3)(\sqrt{x+1}+2)}$
$= \lim_{x\to 3}\frac{x-3}{(x-3)(\sqrt{x+1}+2)}$
$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2} = \frac{1}{4}$

49. $\lim_{x\to 0} x = \frac{\sqrt{x+5}-\sqrt{5}}{x} = \lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}\cdot\frac{\sqrt{x+5}+\sqrt{5}}{\sqrt{x+5}+\sqrt{5}}$
$= \lim_{x\to 0}\frac{(x+5)-5}{x(\sqrt{x+5}+\sqrt{5})}$
$= \lim_{x\to 0}\frac{x}{x(\sqrt{x+5}+\sqrt{5})}$
$= \lim_{x\to 0}\frac{1}{\sqrt{x+5}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$

50. $\lim_{x\to 0} x = \dfrac{\sqrt{x+2}-\sqrt{2}}{x} = \lim_{x\to 0} \dfrac{\sqrt{x+2}-\sqrt{2}}{x} \cdot \dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}$

$= \lim_{x\to 0} \dfrac{(x+2)-2}{x(\sqrt{x+2}+\sqrt{2})}$

$= \lim_{x\to 0} \dfrac{x}{x(\sqrt{x+2}+\sqrt{2})}$

$= \lim_{x\to 0} \dfrac{1}{\sqrt{x+2}+\sqrt{2}} = \dfrac{1}{2\sqrt{2}}$

51. $\lim_{x\to 2^-}(4-x) = 2$

$\lim_{x\to 2^+}(4-x) = 2$

So, $\lim_{x\to 2} f(x) = 2$

52. $\lim_{x\to 1^-}(x^2+2) = 3$

$\lim_{x\to 1^+}(x^2+2) = 3$

So, $\lim_{x\to 1} f(x) = 3$

53. $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}\left(\frac{1}{3}x - 5\right) = -4$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(-3x+7) = -2$

So, $\lim_{x\to 3} f(x)$ does not exist.

54. $\lim_{s\to 4^-} f(s) = \lim_{s\to 4^-}(3s-4) = 8$

$\lim_{s\to 4^+} f(s) = \lim_{s\to 4^+}\left(5 - \frac{1}{2}s\right) = 3$

So, $\lim_{s\to 4} f(s)$ does not exist.

55. $\lim_{x\to -4} \dfrac{2}{x+4} = \dfrac{2}{0}$

The limit does not exist.

56. $\lim_{x\to 5} \dfrac{4}{x-5} = \dfrac{4}{0}$

The limit does not exist.

57. $\lim_{x\to 2} \dfrac{x-2}{x^2-4x+4} = \lim_{x\to 2} \dfrac{x-2}{(x-2)(x-2)}$

$= \lim_{x\to 2} \dfrac{1}{x-2}$

The limit does not exist.

58. $\lim_{t\to -6} \dfrac{t+6}{t^2+12t+36} = \lim_{t\to -6} \dfrac{t+6}{(t+6)(t+6)}$

$= \lim_{t\to -6} \dfrac{1}{t+6} = \dfrac{1}{0}$

The limit does not exist.

59. (a) $\lim_{x\to 3^+} f(x) = 1$

(b) $\lim_{x\to 3^-} f(x) = 1$

(c) $\lim_{x\to 3} f(x) = 1$

60. (a) $\lim_{x\to -2^+} f(x) = -2$

(b) $\lim_{x\to -2^-} f(x) = -2$

(c) $\lim_{x\to -2} f(x) = -2$

61. (a) $\lim_{x\to 2^-} f(x) = -1$

(b) $\lim_{x\to 2^+} f(x) = -1$

(c) $\lim_{x\to 2} f(x) = -1$

62. (a) $\lim_{x\to 1^-} f(x) = 3$

(b) $\lim_{x\to 1^+} f(x) = 3$

(c) $\lim_{x\to 1} f(x) = 3$

63. (a) $\lim_{x\to 6^-} f(x) = -6$

(b) $\lim_{x\to 6^+} f(x) = 6$

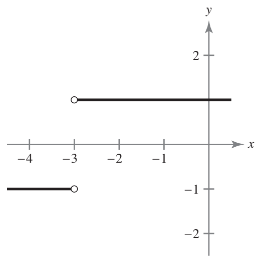
(c) $\lim_{x\to 6} f(x)$ does not exist.

64. (a) $\lim_{x\to -1^+} f(x) = 0$

(b) $\lim_{x\to -1^-} f(x) = 2$

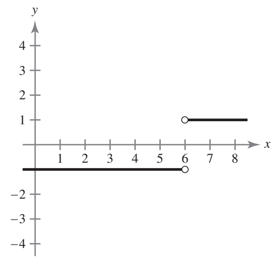
(c) $\lim_{x\to -1} f(x)$ does not exist.

65.



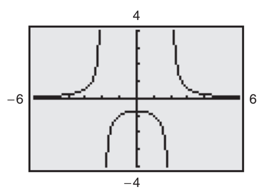
$\lim_{x\to -3^-} \frac{|x+3|}{x+3} = -1$ and $\lim_{x\to -3^+} \frac{|x+3|}{x+3} = 1$

66.



$\lim_{x\to 6^-} \frac{|x-6|}{x-6} = -1$ and $\lim_{x\to 6^+} \frac{|x-6|}{x-6} = 1$

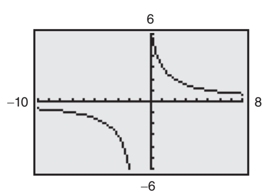
67.



x	3	2.5	2.1	2.01	2.001	2.0001	2
$f(x)$	0.6	1.33	7.32	74.81	749.81	7499.81	Undefined

The limit does not exist because f is unbounded as x approaches 2 from the right.

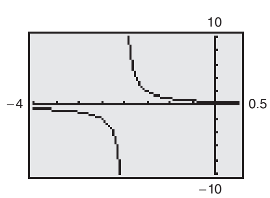
68.



x	−1	−0.999	−0.99	−0.9	−0.5	0	1
$f(x)$	Undefined	6000	600	60	12	6	3

Because $f(x) = \frac{6}{x+1}$ decreases without bound as x approaches -1 from the right, the limit does not exist.

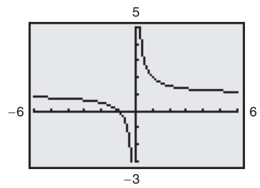
69.



x	−3	−2.5	−2.1	−2.01	−2.001	−2.0001	−2
$f(x)$	−1	−2	−10	−100	−1000	−10,000	undefined

Because $f(x) = \frac{1}{x+2}$ decreases without bound as x approaches -2 from the left, the limit does not exist.

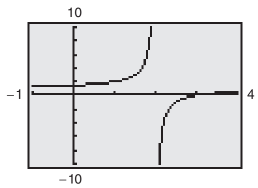
70.



x	−1	−0.5	−0.1	−0.01	−0.001	−0.0001	0
$f(x)$	0	−1	−9	−99	−999	−9999	undefined

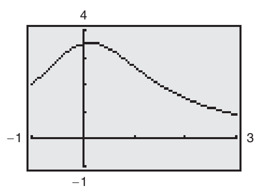
Because $f(x) = \dfrac{x+1}{x}$ decreases without bound as x approaches 0 from the left, the limit does not exist.

71.



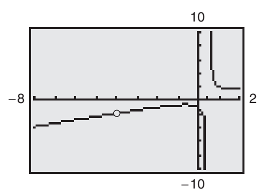
$\lim_{x \to 2} \dfrac{x^2 - 5x + 6}{x^2 - 4x + 4}$ does not exist.

72.



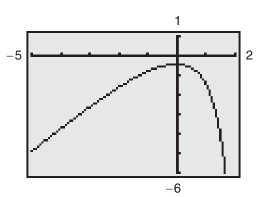
$$\lim_{x \to 1} \frac{x^2 + 6x - 7}{x^3 - x^2 + 2x - 2} \approx 2.667$$

73.



$$\lim_{x \to -4} \frac{x^3 + 4x^2 + x + 4}{2x^2 + 7x - 4} \approx -1.889$$

74.



$$\lim_{x \to -2} \frac{4x^3 + 7x^2 + x + 6}{3x^2 - x - 14} \approx -1.615$$

75. $C = \dfrac{25p}{100 - p}, \ 0 \le p < 100$

(a) $C(50) = \dfrac{25(50)}{100 - 50} = \$25$ thousand

(b) Find p for $C = 100$.

$$\begin{aligned} 100 &= \frac{25p}{100 - p} \\ 100(100 - p) &= 25p \\ 10{,}000 - 100p &= 25p \\ 10{,}000 &= 125p \\ 80 &= p, \text{ or } 80\% \end{aligned}$$

(c) $\lim_{p \to 100^-} = \lim_{p \to 100^-} \dfrac{25p}{100 - p} = \infty$

The cost function increases without bound as x approaches 100 from the left. Therefore, according to the model, it is not possible to remove 100% of the pollutants.

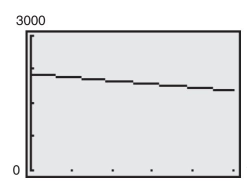
76. (a) $\lim_{x \to 50} C$ does not exist. The two one-sided limits are not equal.

$\lim_{x \to 50^-} C \approx \$7.50$ and $\lim_{x \to 50^+} C \approx \$5$

(b) $\lim_{x \to 150} C \approx \$10.50$

(c) It would be less expensive to make 201 copies, since $\lim_{x \to 200^-} C \approx \$14$ and $\lim_{x \to 200^+} C \approx \$10$.

77. (a)



(b) When $x = 0.25$: $A = 2685.06$

When $x = \frac{1}{365}$: $A = 2717.91$

(c) Using the *zoom* and *trace* features, $\lim_{x \to 0^+} A \approx \$2718.28$. Because x, the length of the compounding period, is approaching 0, this limit represents the balance with continuous compounding.

Section 1.6 Continuity

Skills Warm Up

1. $\dfrac{x^2+6x+8}{x^2-6x-16} = \dfrac{(x+4)(x+2)}{(x-8)(x+2)} = \dfrac{x+4}{x-8}$

2. $\dfrac{x^2-5x-6}{x^2-9x+18} = \dfrac{(x-6)(x+1)}{(x-6)(x-3)} = \dfrac{x+1}{x-3}$

3. $\dfrac{2x^2-2x-12}{4x^2-24x+36} = \dfrac{2(x^2-x-6)}{4(x^2-6x+9)}$
$= \dfrac{2(x-3)(x+2)}{4(x-3)(x-3)}$
$= \dfrac{x+2}{2(x-3)}$

4. $\dfrac{x^3-16x}{x^3+2x^2-8x} = \dfrac{x(x^2-16)}{x(x^2+2x-8)}$
$= \dfrac{x(x^2-16)}{x(x+4)(x-2)}$
$= \dfrac{x(x+4)(x-4)}{x(x+4)(x-2)}$
$= \dfrac{x-4}{x-2}$

5. $x^2+7x = 0$
$x(x+7) = 0$
$x = 0$
$x+7 = 0 \Rightarrow x = -7$

6. $x^2+4x-5 = 0$
$(x+5)(x-1) = 0$
$x+5 = 0 \Rightarrow x = -5$
$x-1 = 0 \Rightarrow x = 1$

7. $3x^2+8x+4 = 0$
$(3x+2)(x+2) = 0$
$3x+2 = 0 \Rightarrow x = -\frac{2}{3}$
$x+2 = 0 \Rightarrow x = -2$

8. $3x^3-x^2-24x = 0$
$x(3x^2-x-24) = 0$
$x(3x+8)(x-3) = 0$
$x = 0$
$3x+8 = 0 \Rightarrow x = -\frac{8}{3}$
$x-3 = 0 \Rightarrow x = 3$

9. $\lim_{x\to 3}(2x^2-3x+4) = 2(3^2)-3(3)+4$
$= 2(9)-9+4$
$= 13$

10. $\lim_{x\to -2}\sqrt{x^2-x+3} = \sqrt{(-2)^3-(-2)+3}$
$= 3$

1. Continuous; The function is a polynomial.

2. Continuous; The function is a polynomial.

3. Not continuous; The rational function is not defined at $x = \pm 4$.

4. Not continuous; The rational function is not defined at $x = \pm 3$.

5. Continuous; The rational function's domain is the entire real line.

6. Continuous; The rational function's domain is the entire real line.

7. Not continuous; The rational function is not defined at $x = 3$ or $x = 5$.

8. Not continuous; The rational function is not defined at $x = 1$ or $x = 5$.

9. Not continuous; The rational function is not defined at $x = \pm 6$.

10. Not continuous; The rational function is not defined at $x = \pm 5$.

11. $f(x) = \dfrac{x^2-1}{x}$ is continuous on $(-\infty, 0)$ and $(0, \infty)$ because the domain of f consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined and $\lim_{x\to 0} f(x)$ does not exist.

12. $f(x) = \dfrac{1}{x^2 - 4}$ is continuous on $(-\infty, -2)$, $(-2, 2)$ and $(2, \infty)$ because the domain of f consists of all real numbers except $x = \pm 2$. There are discontinuities at $x = \pm 2$ because $f(2)$ and $f(-2)$ are not defined and $\lim_{x \to 2} f(x)$ and $\lim_{x \to -2} f(x)$ do not exist.

13. $f(x) = \dfrac{x^2 - 1}{x + 1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined and $\lim_{x \to -1} f(x) \neq f(-1)$.

14. $f(x) = \dfrac{x^3 - 27}{x - 3}$ is continuous on $(-\infty, 3)$ and $(3, \infty)$ because the domain of f consists of all real numbers except $x = 3$. There is a discontinuity at $x = 3$ because $f(3)$ is not defined and $\lim_{x \to 3} f(x) \neq f(3)$.

15. $f(x) = x^2 - 9x + 14$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

16. $f(x) = 3 - 2x - x^2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

17. $f(x) = \dfrac{x}{x^2 - 1} = \dfrac{x}{(x + 1)(x - 1)}$ is continuous on $(-\infty, -1)$, $(-1, 1)$, and $(1, \infty)$ because the domain of f consists of all real numbers except $x = \pm 1$. There are discontinuities at $x = \pm 1$ because $f(1)$ and $f(-1)$ are not defined and $\lim_{x \to 1} f(x)$ and $\lim_{x \to -1} f(x)$ do not exist.

18. $f(x) = \dfrac{x - 3}{x^2 - 9}$ is continuous on $(-\infty, -3)$, $(-3, 3)$, and $(3, \infty)$ because the domain of f consists of all real numbers except $x = \pm 3$. There are discontinuities at $x = \pm 3$ because $f(-3)$ and $f(3)$ are not defined, $\lim_{x \to -3}$ does not exist, and $\lim_{x \to 3} f(x) \neq f(3)$.

19. $f(x) = \dfrac{7x}{x^2 + 5}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

20. $f(x) = \dfrac{6}{x^2 + 3}$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real numbers.

21. $f(x) = \dfrac{x - 5}{x^2 - 9x + 20} = \dfrac{x - 5}{(x - 5)(x - 4)}$ is continuous on $(-\infty, 4)$, $(4, 5)$, and $(5, \infty)$ because the domain of f consists of all real numbers except $x = 4$ and $x = 5$. There is a discontinuity at $x = 4$ and $x = 5$ because $f(4)$ and $f(5)$ are not defined and $\lim_{x \to 4} f(x)$ does not exist and $\lim_{x \to 5} f(x) \neq f(5)$.

22. $f(x) = \dfrac{x - 1}{x^2 + x - 2} = \dfrac{x - 1}{(x - 1)(x + 2)}$ is continuous on $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$ because the domain of f consists of all real number except $x = -2$ and $x = 1$. There is discontinuity at $x = -2$ and $x = 1$ because $f(-2)$ and $f(1)$ are not defined, $\lim_{x \to -2} f(x)$ does not exist, and $\lim_{x \to 1} f(x) \neq f(1)$.

23. $f(x) = \sqrt{4 - x}$ is continuous on $(-\infty, 4]$ because the domain of f consists of all real $x \leq 4$.

24. $f(x) = \sqrt{x - 1}$ is continuous on $[1, \infty)$ because the domain of f consists of all real $x \geq 1$.

25. $f(x) = \sqrt{x} + 2$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \geq 0$.

26. $f(x) = 3 - \sqrt{x}$ is continuous on $[0, \infty)$ because the domain of f consists of all real $x \geq 0$.

27. $f(x) = \begin{cases} -2x + 3, & -1 \leq x \leq 1 \\ x^2, & 1 < x \leq 3 \end{cases}$ is continuous on $[-1, 3]$.

28. $f(x) = \begin{cases} \frac{1}{2}x + 1, & -3 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 4 \end{cases}$ is continuous on $[-3, 2)$, $(2, 4]$. f is discontinuous at $x = 2$ because $\lim_{x \to 2} f(x)$ does not exist.

$\lim_{x \to 2^-} f(x) = 2$ and $\lim_{x \to 2^+} f(x) = 1$.

29. $f(x) = \begin{cases} 4 - 2x, & x \leq 2 \\ x^2 - 3, & x > 2 \end{cases}$

is continuous on $(-\infty, 2)$ and $(2, \infty)$. There is a discontinuity at $x = 2$ because $\lim_{x \to 2} f(x)$ does not exist.

30. $f(x) = \begin{cases} x^2 - 2, & x \le -1 \\ 3x + 2, & x > -1 \end{cases}$ is continuous on $(-\infty, \infty.)$

31. $f(x) = \dfrac{|x + 1|}{x + 1}$ is continuous on $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined, and $\lim_{x \to -1} f(x)$ does not exist.

32. $f(x) = \dfrac{|4 - x|}{4 - x}$ is continuous on $(-\infty, 4)$ and $(4, \infty)$ because the domain of f consists of all real numbers except $x = 4$. There is a discontinuity at $x = 4$ because $f(4)$ is not defined and $\lim_{x \to 4} f(x)$ does not exist.

33. $f(x) = x\sqrt{x + 3}$ is continuous on $[-3, \infty)$.

34. $f(x) = \dfrac{x + 1}{\sqrt{x}}$ is continuous on $(0, \infty)$.

35. $f(x) = [\![2x]\!] + 1$ is continuous on all intervals of the form $\left(\frac{1}{2}c, \frac{1}{2}c + \frac{1}{2}\right)$, where c is an integer. That is, f is continuous on $\ldots, \left(-\frac{1}{2}, 0\right), \left(0, \frac{1}{2}\right), \left(\frac{1}{2}, 1\right), \ldots$. f is not continuous at all points $\frac{1}{2}c$, where c is an integer. There are discontinuities at $x = \dfrac{c}{2}$, where c is an integer, because $\lim_{x \to c/2} f(x)$ does not exist.

36. $f(x) = \dfrac{[\![x]\!]}{2} + x$ is continuous on all intervals of the form $(c, c + 1)$ where c is an integer. There are discontinuities at all integer values c because $\lim_{x \to c} f(x)$ does not exist.

37. $f(x) = [\![x - 1]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at $x = c$, where c is an integer, because $\lim_{x \to c} f(x)$ does not exist.

38. $f(x) = x - [\![x]\!]$ is continuous on all intervals $(c, c + 1)$. There are discontinuities at all integer values c because $\lim_{x \to c} f(x)$ does not exist.

39. $h(x) = f(g(x)) = f(x - 1) = \dfrac{1}{\sqrt{x - 1}}, \; x > 1$

h is continuous on its entire domain $(1, \infty)$.

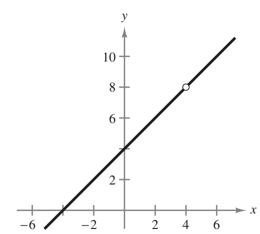
40. $h(x) = f(g(x)) = f(x^2 + 5)$

$$= \frac{1}{(x^2 + 5) - 1} = \frac{1}{x^2 + 4}$$

h is continuous on $(-\infty, \infty)$.

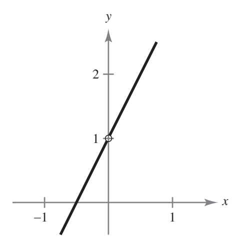
41. $f(x) = \dfrac{x^2 - 16}{x - 4} = \dfrac{(x + 4)(x - 4)}{x - 4} = x + 4, \; x \ne 4$

f has a removable discontinuity at $x = 4$;
Continuous on $(-\infty, 4)$ and $(4, \infty)$.



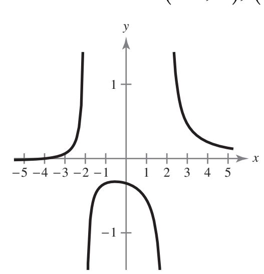
42. $f(x) = \dfrac{2x^2 + x}{x} = \dfrac{x(2x + 1)}{x}$

f has a removable discontinuity at $x = 0$;
Continuous on $(-\infty, 0)$ and $(0, \infty)$.



43. $f(x) = \dfrac{x + 4}{3x^2 - 12}$

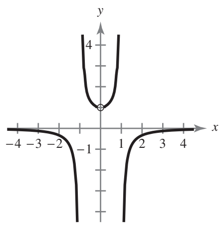
Continuous on $(-\infty, 2)$, $(-2, 2)$, and $(2, \infty)$.



44. $f(x) = \dfrac{-x}{x^3 - x} = \dfrac{-x}{x(x+1)(x-1)} = -\dfrac{1}{(x+1)(x-1)},\ x \neq 0$

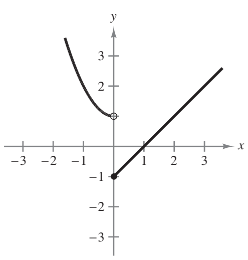
f has a removable discontinuity at $x = 0$; *f* has nonremovable discontinuities at $x = -1$ and $x = 1$;

Continuous on $(-\infty, -1)$, $(-1, 0)$, $(0, 1)$, and $(1, \infty)$.



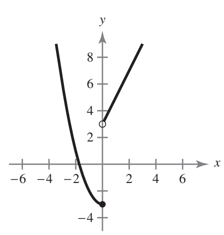
45. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$;
Continuous on $(-\infty, 0)$ and $(0, \infty)$.



46. $f(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 2x + 3, & x > 0 \end{cases}$

f has a nonremovable discontinuity at $x = 0$; Continuous on $(-\infty, 0)$ and $(0, \infty)$.



47. Continuous on $[-1, 5]$ because $f(x) = x^2 - 4x - 5$ is a polynomial.

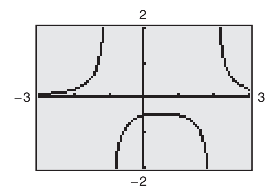
48. Continuous on $[-2, 2]$ because $f(x) = \dfrac{5}{x^2 + 1}$ is defined on the entire interval.

49. Continuous on $[1, 2)$ and $(2, 4]$ because $f(x) = \dfrac{1}{x - 2}$ has a nonremovable discontinuity at $x = 2$.

50. Continuous on $[0, 1)$, $(1, 3)$, and $(3, 4]$ because

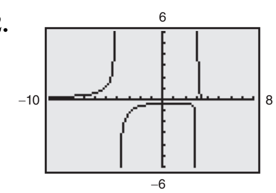
$f(x) = \dfrac{x - 1}{(x-1)(x-3)} = \dfrac{1}{x - 3},\ x \neq 1$, has a removable discontinuity at $x = 1$ and a nonremoveable discontinuity at $x = 3$.

51.



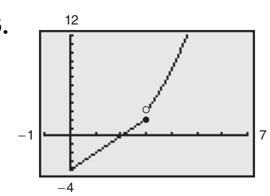
From the graph, you can see that $h(2)$ and $h(-1)$ are not defined, so *h* is not continuous at $x = 2$ and $x = -1$.

52.



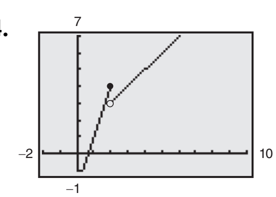
From the graph, you can see that $k(-3)$ is not defined, so *k* is not continuous at $x = -3$. [**Note:** There is a hole at $x = 4$.]

53.

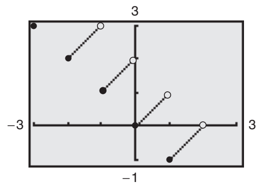


From the graph, you can see that $\lim_{x \to 3} f(x)$ does not exist, so *f* is not continuous at $x = 3$.

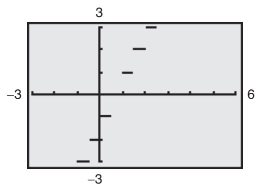
54.



From the graph, you can see that $\lim_{x \to 2} f(x)$ does not exist, so *f* is not continuous at $x = 2$.

55. 

From the graph, you can see that $\lim_{x \to c} (x - 2[\![x]\!])$, where c is an integer, does not exist. So f is not continuous at all integers c.

56. 

From the graph, you can see that $\lim_{x \to c/2} [\![2x - 1]\!]$, where c is an integer, does not exist. So f is not continuous at all integers c.

57. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^3 = 8$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} ax^2 = 4a$

So, $8 = 4a$ and $a = 2$.

58. $\lim_{x \to -1^-} f(x) = 2$

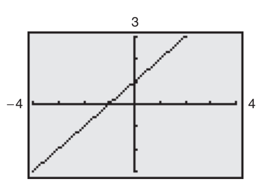
$\lim_{x \to -1^+} f(x) = -a + b$

$\lim_{x \to 3^-} f(x) = 3a + b$

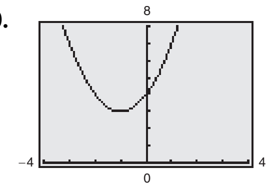
$\lim_{x \to 3^+} f(x) = -2$

So,

$$\begin{aligned} -a + b &= 2 \\ 3a + b &= -2 \\ \hline -4a \quad &= 4 \\ a &= -1 \\ b &= 1. \end{aligned}$$

59. 

$f(x) = \dfrac{x^2 + x}{x} = \dfrac{x(x + 1)}{x}$ appears to be continuous on $[-4, 4]$. But it is not continuous at $x = 0$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

60. 

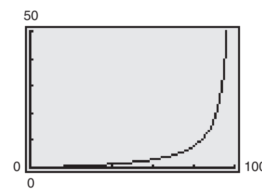
$f(x) = \dfrac{x^3 - 8}{x - 2} = \dfrac{(x - 2)(x^2 + 2x + 4)}{(x - 2)}$ appears to be continuous on $[-4, 4]$. But, it is not continuous at $x = 2$ (removable discontinuity). Examining a function analytically can reveal removable discontinuities that are difficult to find just from analyzing its graph.

61. (a) $[0, 100)$; Negative x-values and values greater than 100 do not make sense in this context. Also, $C(100)$ is undefined.

(b) C is continuous on its domain because all rational functions are continuous on their domains.

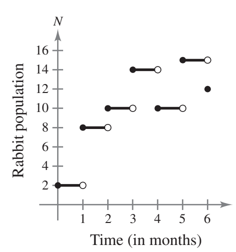
(c) For $x = 75$,

$$C = \frac{2(75)}{100 - 75} = \frac{150}{25} = 6 \text{ million dollars.}$$



62. (a) The graph of G is not continuous on day 8 and day 22.

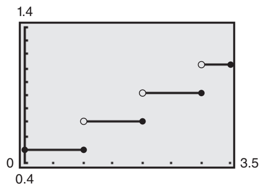
(b) On these days, the person fills his or her gas tank.

63. 

There are nonremovable discontinuities at $t = 1$, 2, 3, 4, 5, and 6.

64. Yes, a linear model is a continuous function. No, actual revenue would probably not be continuous because revenue is usually recorded over larger units of time (hourly, daily, or monthly). In these cases, the revenue may jump between different units of time.

65. (a)

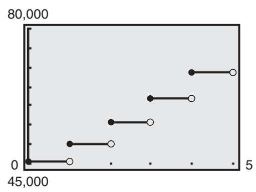


There are nonremovable discontinuities at $x = 1, 2,$ and 3. Explanations will vary.

(b) $C(2.5) = \$0.91$

A 2.5-ounce letter costs \$0.91.

66. (a)



Nonremovable discontinuities at $t = 1, 2, 3, 4, 5$

S is not continuous at $t = 1, 2, 3, 4,$ or 5.

(b) For $t = 5$, $S(5) = 45{,}300(1.11)^{[\![5]\!]} = 76{,}333.13$.

The salary during the fifth year is \$76,333.13.

67. (a) The graph has nonremovable discontinuities at $t = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$.

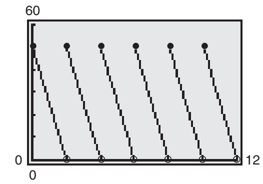
(b) Let $t = 2$.

$A = 7500(1.015)^{[\![4 \cdot 2]\!]} \approx \$8448.69$

(c) Let $t = 7$.

$A = 7500(1.015)^{[\![4 \cdot 7]\!]} \approx \$11{,}379.17$

68. (a)



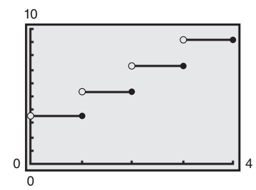
Nonremovable discontinuities at $t = 2, 4, 6, 8, \ldots$; N is not continuous at $t = 2, 4, 6, 8, \ldots$.

(b) For $t = 7$, $N = 25\left(2\left[\!\!\left[\frac{7+2}{2}\right]\!\!\right] - 7\right) = 25$. During the seventh month, there are 25 units in inventory.

(c) $N \to 0$ when $t \to 2^-, 4^-, 6^-, 8^-, \ldots$, so the inventory is replenished every two months.

69. (a) $C(x) = 3.50 - 1.90[\![1 - x]\!], x > 0$

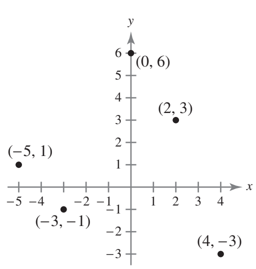
(b)



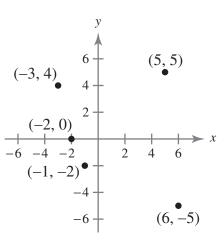
C is not continuous at all integers.

Review Exercises for Chapter 1

1.



2.



3. $\text{Distance} = \sqrt{(0-5)^2 + (0-2)^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$

4. $\text{Distance} = \sqrt{(3-0)^2 + (4-2)^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$

5. $\text{Distance} = \sqrt{[-1 - (-4)]^2 + (3-6)^2}$

$= \sqrt{9 + 9}$

$= 3\sqrt{2}$

6. $\text{Distance} = \sqrt{[(-2) - (-8)]^2 + (7-5)^2}$

$= \sqrt{36 + 4}$

$= \sqrt{40} = 2\sqrt{10}$

7. $d = \sqrt{\left(\frac{3}{4} - \frac{1}{4}\right)^2 + \left(-6 - (-8)\right)^2}$

$= \sqrt{\left(\frac{1}{2}\right)^2 + (2)^2} = \sqrt{\frac{1}{4} + 4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$

8. $d = \sqrt{\left(4 - (-0.6)\right)^2 + (-1.8 - 3)^2}$

$= \sqrt{(4.6)^2 + (-4.8)^2} = \sqrt{21.16 + 23.04} = \sqrt{44.2} \approx 6.65$

9. Midpoint $= \left(\dfrac{5 + 9}{2}, \dfrac{6 + 2}{2}\right) = (7, 4)$

10. Midpoint $= \left(\dfrac{-7 + 3}{2}, \dfrac{0 + 6}{2}\right) = (-2, 3)$

11. Midpoint $= \left(\dfrac{-10 - 6}{2}, \dfrac{4 + 8}{2}\right) = (-8, 6)$

12. Midpoint $= \left(\dfrac{7 - 3}{2}, \dfrac{-9 + 5}{2}\right) = (2, -2)$

13. Midpoint $= \left(\dfrac{-2 + 4.3}{2}, \dfrac{0.1 + (-3)}{2}\right)$

$= \left(\dfrac{2.3}{2}, \dfrac{-2.9}{5}\right)$

$= (1.15, -1.45)$

14. Midpoint $= \left(\dfrac{\frac{1}{2} + 1}{2}, \dfrac{\frac{5}{2} + \left(-\frac{3}{4}\right)}{2}\right)$

$= \left(\dfrac{\frac{3}{2}}{2}, \dfrac{\frac{7}{4}}{5}\right)$

$= \left(\dfrac{3}{4}, \dfrac{7}{20}\right)$

15. $P = R - C$. The tallest bars represent revenues. The middle bars represent costs. The bars on the left of each group represent profits because $P = R - C$.

16. 2009: $R \approx$ \$24.0 billion
$C \approx$ \$16.0 billion
$P \approx$ \$8.0 billion

2010: $R \approx$ \$30.0 billion
$C \approx$ \$22.0 billion
$P \approx$ \$8.0 billion

2011: $R \approx$ \$38.0 billion
$C \approx$ \$30.0 billion
$P \approx$ \$8.0 billion

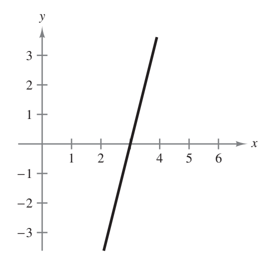
2012: $R \approx$ \$50.0 billion
$C \approx$ \$40.0 billion
$P \approx$ \$10.0 billion

2013: $R \approx$ \$58.0 billion
$C \approx$ \$48.0 billion
$P \approx$ \$10.0 billion

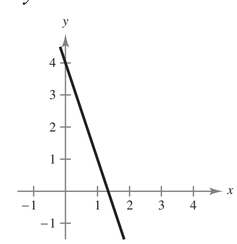
17. $(1, 3)$ translates to $(3, 6)$.
$(2, 4)$ translates to $(4, 7)$.
$(4, 1)$ translates to $(6, 4)$.

18. $(-2, 1)$ translates to $(-7, -1)$.
$(-1, 2)$ translates to $(-6, 0)$.
$(1, 0)$ translates to $(-4, -2)$.
$(0, -1)$ translates to $(-5, -3)$.

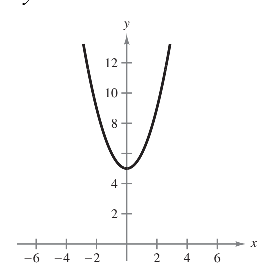
19. $y = 4x - 12$



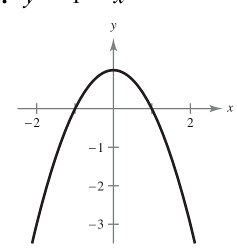
20. $y = 4 - 3x$



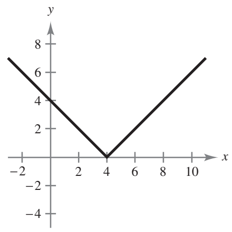
21. $y = x^2 + 5$



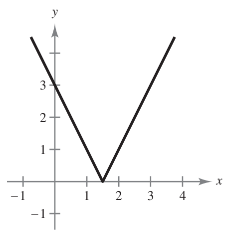
22. $y = 1 - x^2$



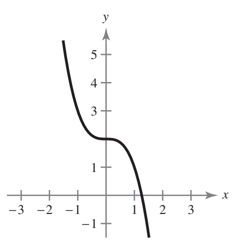
23. $y = |4 - x|$



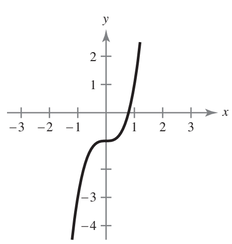
24. $y = |2x - 3|$



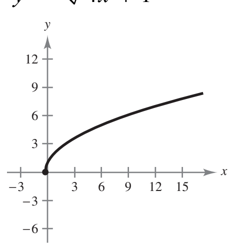
25. $y = 2 - x^3$



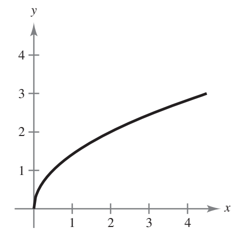
26. $y = 2x^3 - 1$



27. $y = \sqrt{4x + 1}$



28. $y = \sqrt{2x}$



29. Let $y = 0$. Then,

$$4x + 0 + 3 = 0$$
$$x = -\tfrac{3}{4}.$$

Let $x = 0$. Then,

$$4(0) + y + 3 = 0$$
$$y = -3.$$

x-intercept: $\left(-\tfrac{3}{4}, 0\right)$

y-intercept: $(0, -3)$

30. Let $y = 0$. Then,

$$3x - (0) + 6 = 0$$
$$3x = -6$$
$$x = -2.$$

Let $x = 0$. Then,

$$3(0) - y + 6 = 0$$
$$-y = -6$$
$$y = 6.$$

x-intercept: $(-2, 0)$

y-intercept: $(0, 6)$

31. Let $y = 0$. Then,

$$0 = x^2 + 2x - 8$$
$$0 = (x + 4)(x - 2)$$
$$x + 4 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -4 \qquad\qquad x = 2.$$

Let $x = 0$. Then,

$$y = (0)^2 + 2(0) - 8$$
$$y = -8.$$

x-intercepts: $(-4, 0)$, $(2, 0)$

y-intercept: $(0, -8)$

32. Let $y = 0$. Then,

$$0 = (x - 1)^3 + 2(x - 1)^2$$
$$0 = (x - 1)^2(x + 1)$$
$$x = \pm 1.$$

Let $x = 0$. Then,

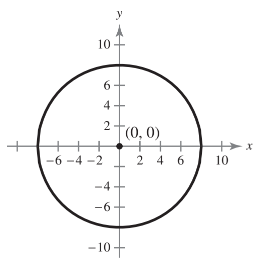
$$y = (0 - 1)^3 + 2(0 - 1)^2$$
$$y = 1.$$

x-intercepts: $(-1, 0), (1, 0)$

y-intercept: $(0, 1)$

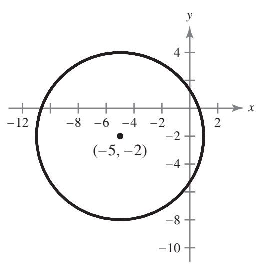
33. $(x - 0)^2 + (y - 0)^2 = 8^2$

$$x^2 + y^2 = 64$$



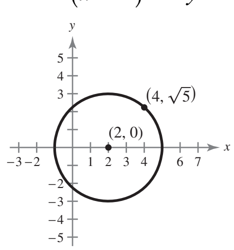
34. $(x - (-5))^2 + (y - (-2))^2 = 6^2$

$$(x + 5)^2 + (y + 2)^2 = 36$$



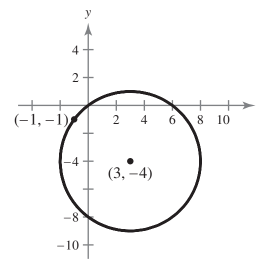
35. $(x - 2)^2 + (y - 0)^2 = r^2$

$$(x - 2)^2 + y^2 = r^2$$
$$(4 - 2)^2 + (\sqrt{5})^2 = r^2$$
$$4 + 5 = r^2$$
$$9 = r^2$$
$$(x - 2)^2 + y^2 = 9$$



36. $(x - 3)^2 + (y - (-4))^2 = r^2$

$$(x - 3)^2 + (y + 4)^2 = r^2$$
$$(-1 - 3)^2 + (-1 + 4)^2 = r^2$$
$$(-4)^2 + (3)^2 = r^2$$
$$16 + 9 = r^2$$
$$25 = r^2$$
$$(x - 3)^2 + (y + 4)^2 = 25$$



37. $y = 2x + 13$ and $y = -5x - 1$

Set the two equations equal to each other.

$$2x + 13 = -5x - 1$$
$$7x = -14$$
$$x = -2$$

Substitute $x = -2$ into one of the equations.

$$y = 2(-2) + 13 = 9$$

Point of intersection: $(-2, 9)$

38. $y = x^2 + 3$ and $y = 9 - x$

Set the two equations equal to each other.

$$x^2 + 3 = 9 - x$$
$$x^2 + x - 6 = 0$$
$$(x + 3)(x - 2) = 0$$
$$x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$
$$x = -3 \qquad x = 2$$

Substitute $x = -3$ and $x = 2$ into one of the equations.

$$y = (-3)^2 + 3 = 12$$
$$y = (2)^2 + 3 = 7$$

Points of intersection: $(-3, 12)$ and $(2, 7)$

39. By equating the y-values for the two equations, you have

$$x^3 = x$$
$$x(x^2 - 1) = 0$$
$$x = -1, 0, 1.$$

The corresponding y-values are $y = -1$, $y = 0$, and $y = 1$, so the points of intersection are $(-1, -1)$, $(0, 0)$, and $(1, 1)$.

40. $y = x^3 + 4x^2 - 3$ and $y = -2x^2 + 27x - 3$

Set the two equations equal to each other.

$$x^3 + 4x^2 - 3 = -2x^2 + 27x - 3$$
$$x^3 + 6x^2 - 27x = 0$$
$$x(x^2 + 6x - 27) = 0$$
$$x(x + 9)(x - 3) = 0$$
$$x = 0$$
$$x + 9 = 0 \rightarrow x = -9$$
$$x - 3 = 0 \rightarrow x = 3$$

Substitute $x = 0$, $x = -9$, and $x = 3$ into one of the equations.

$$y = (0)^3 + 4(0)^2 - 3 = -3$$
$$y = (-9)^3 + 4(-9)^2 - 3 = -408$$
$$y = (3)^3 + 4(3)^2 - 3 = 60$$

Points of intersection: $(0, -3)$, $(-9, -408)$, and $(3, 60)$

41. (a) $C = 200 + 2x + 8x = 200 + 10x$

$R = 14x$

(b)
$$C = R$$
$$200 + 10x = 14x$$
$$200 = 4x$$
$$x = 50 \text{ shirts}$$

$(x, R) = (x, C) = (50, 700)$

(c) $P = R - C$

$P = 14x - (200 + 10x)$

$P = 4x - 200$

To find the number of shirts that yields a profit of \$600, set $P = 600$ and solve for x.

$$600 = 4x - 200$$
$$800 = 4x$$
$$200 = x$$

So, 200 shirts will yield a profit of \$600.

42. (a) $C = 6000 + 6.50x$

$R = 13.90x$

(b)
$$C = R$$
$$6000 + 6.5x = 13.9x$$
$$6000 = 7.4x$$
$$x \approx 810.81, \text{ or } 811 \text{ units}$$

(c) $P = R - C$

$P = 13.9x - (6000 + 6.5x)$

$P = 7.4x - 6000$

To find the number of units that yields a profit of \$1500, set $P = 1500$ and solve for x.

$$1500 = 7.4x - 6000$$
$$7500 = 7.4x$$
$$x \approx 1014 \text{ units}$$

So, about 1014 units will yield a profit of \$1500.

43. $p = 91.4 - 0.009x = 6.4 + 0.008x$

$$85 = 0.017x$$
$$x = 5000 \text{ units}$$

Equilibrium point $(x, p) = (5000, 46.40)$

44. (a)

Year	2008	2009	2010	2011	2012	2013
Wind (actual)	546	721	923	1168	1340	1595
Wind (model)	542.0	727.2	928.0	1140.7	1361.5	1586.6

The model fits the data well for the years 2008 through 2013.

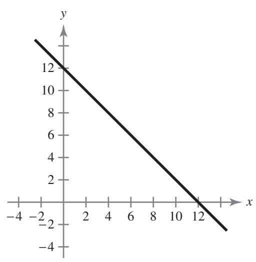
(b) Let $t = 19$.

$y \approx 2815.7$ trillion Btu

45. $y = -x + 12$

Slope: $m = -1$

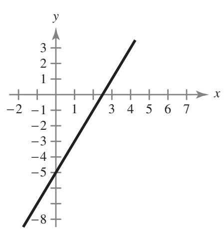
y-intercept: $(0, 12)$



46. $y = 2x - 5$

Slope: $m = 2$

y-intercept: $(0, -5)$

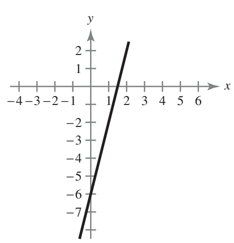


47. $4x - y = 6$

$y = 4x - 6$

Slope: $m = 4$

y-intercept: $(0, -6)$



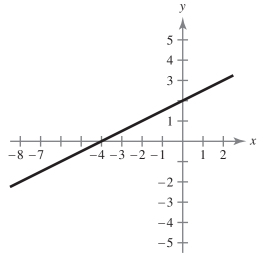
48. $2x - 4y = -8$

$-4y = -2x - 8$

$y = \frac{1}{2}x + 2$

Slope: $m = \frac{1}{2}$

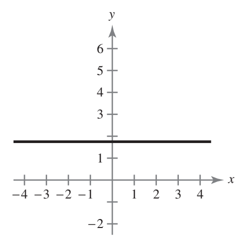
y-intercept: $(0, 2)$



49. $y = \frac{7}{4}$

Slope: $m = 0$ (horizontal line)

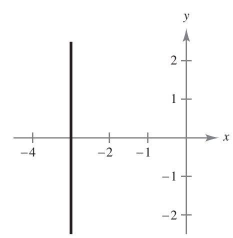
y-intercept: $\left(0, \frac{7}{4}\right)$



50. $x = -3$

Slope: undefined (vertical line)

No y-intercept



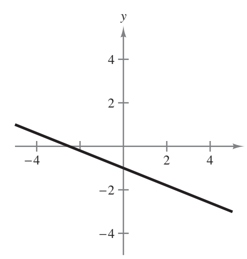
51. $-2x - 5y - 5 = 0$

$5y = -2x - 5$

$y = -\frac{2}{5}x - 1$

Slope: $m = -\frac{2}{5}$

y-intercept: $(0, -1)$



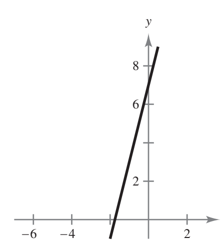
52. $3.2x - 0.8y + 5.6 = 0$

$8y = 32x + 56$

$y = 4x + 7$

Slope: $m = 4$

y-intercept: $(0, 7)$



53. Slope $= \dfrac{6 - 0}{7 - 0} = \dfrac{6}{7}$

54. Slope $= \dfrac{7 - 5}{-5 - (-1)} = -\dfrac{1}{2}$

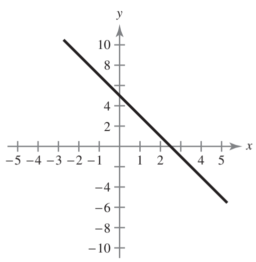
55. Slope $= \dfrac{29 - 29}{-3 - 5} = \dfrac{0}{-8} = 0$

56. Slope $= \dfrac{-3 - (-3)}{-1 - (-11)} = 0$ (horizontal line)

57. $y - (-1) = -2(x - 3)$

$y + 1 = -2x + 6$

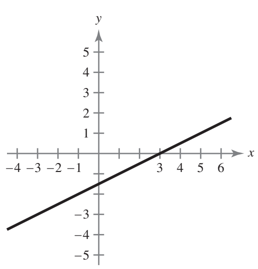
$y = -2x + 5$



58. $y - (-3) = \frac{1}{2}(x - (-3))$

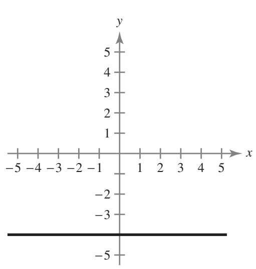
$y + 3 = \frac{1}{2}x + \frac{3}{2}$

$y = \frac{1}{2}x - \frac{3}{2}$



59. $m = 0$: horizontal line through $(1.5, -4)$

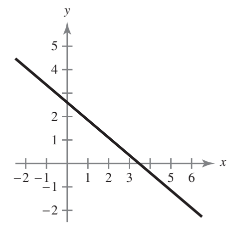
$y = -4$



60. $y - 2 = -\frac{3}{4}\left(x - \frac{4}{5}\right)$

$y - 2 = -\frac{3}{4}x + \frac{3}{5}$

$y = -\frac{3}{4}x + \frac{13}{5}$

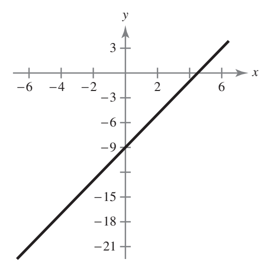


61. $m = \dfrac{5 - (-7)}{7 - 1} = \dfrac{12}{6} = 2$

$y - (-7) = 2(x - 1)$

$y + 7 = 2x - 2$

$y = 2x - 9$



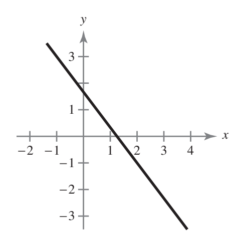
62. $m = \dfrac{-9 - 7}{8 - (-4)} = \dfrac{-16}{12} = -\dfrac{4}{3}$

$y - 7 = -\dfrac{4}{3}(x - (-4))$

$y - 7 = -\dfrac{4}{3}(x + 4)$

$y - 7 = -\dfrac{4}{3}x - \dfrac{16}{3}$

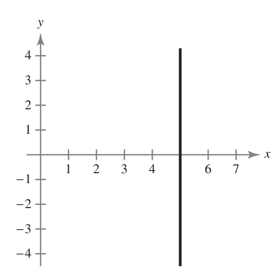
$y = -\dfrac{4}{3}x + \dfrac{5}{3}$



63. $m = \dfrac{14 - 7}{5 - 5} = \dfrac{7}{0} \Rightarrow m$ is undefined.

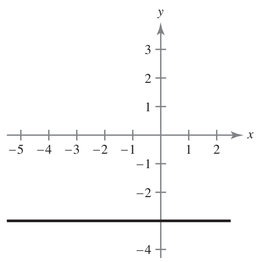
Vertical line through $(5, 7)$

$x = 5$



64. $m = \frac{-3 - (-3)}{-2 - 4} = \frac{0}{-6} = 0 \Rightarrow$ horizontal line through $(4, -3)$

$y = -3$



65. (a)
$$y - 6 = \tfrac{5}{8}[x - (-3)]$$
$$y - 6 = \tfrac{5}{8}(x + 3)$$
$$y - 6 = \tfrac{5}{8}x + \tfrac{15}{8}$$
$$y = \tfrac{5}{8}x + \tfrac{63}{8}$$
$$5x - 8y + 63 = 0$$

(b) $y = -5x - 3$

The given line's slope is $m = -5$, so all the lines perpendicular to the given line have slope $m = \frac{1}{5}$.

$$y - 6 = \tfrac{1}{5}[x - (-3)]$$
$$y - 6 = \tfrac{1}{5}(x + 3)$$
$$y - 6 = \tfrac{1}{5}x + \tfrac{3}{5}$$
$$y = \tfrac{1}{5}x + \tfrac{33}{5}$$
$$x - 5y + 33 = 0$$

(c) $4x + 2y = 7 \Rightarrow y = -2x + \frac{7}{2}$

The given line's slope is $m = -2$, so all the lines parallel to the given line have slope $m = -2$.

$$y - 6 = -2[x - (-3)]$$
$$y = -2x$$
$$2x + y = 0$$

(d) $3x - 2y = 2 \Rightarrow y = \frac{3}{2}x - 1$

The given line's slope is $m = \frac{3}{2}$, so all the lines perpendicular to the given line have slope $m = -\frac{2}{3}$.

$$y - 6 = -\tfrac{2}{3}[x - (-3)]$$
$$y = -\tfrac{2}{3}x + 4$$
$$2x + 3y - 12 = 0$$

66. (a) Slope $= 0$

$$y = -3$$
$$y + 3 = 0$$

(b) Slope undefined

$$x = 1$$
$$x - 1 = 0$$

(c) $-4x + 5y = -3$

$$y = \tfrac{4}{5}x - \tfrac{3}{5}$$
$$y - (-3) = \tfrac{4}{5}(x - 1)$$
$$y = \tfrac{4}{5}x - \tfrac{19}{5}$$
$$-\tfrac{4}{5}x + y + \tfrac{19}{5} = 0$$

(d) $5x - 2y = 3$

$$y = \tfrac{5}{2}x - \tfrac{3}{2}$$

Slope of perpendicular is $-\frac{2}{5}$.

$$y - (-3) = -\tfrac{2}{5}(x - 1)$$
$$y = -\tfrac{2}{5}x - \tfrac{13}{5}$$
$$\tfrac{2}{5}x + y + \tfrac{13}{5} = 0$$

67. $(32, 750), (37, 700)$

$$m = \frac{750 - 700}{32 - 37} = \frac{50}{-5} = -10$$

(a) $x - 750 = -10(p - 32)$

$$x = -10p + 1070$$

(b) If $p = 34.50$, $x = -10(34.50) + 1070 = 725$ units

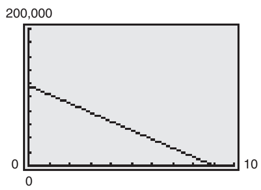
(c) If $p = 42.00$, $x = -10(42.00) + 1070 = 650$ units

68. (0, 117,000), (9, 0)

$$m = \frac{117{,}000}{-9} = -13{,}000$$

(a) $v = -13{,}000(t - 9) = -13{,}000t + 117{,}000$

(b) Graphing utility



(c) $v(4) = \$65{,}000$

(d) $v = 84{,}000$ when $t \approx 2.54$ years

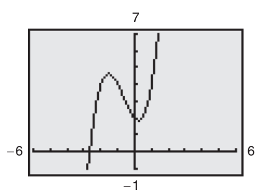
69. Yes, $y = -x^2 + 2$ is a function of x.

70. No, $x = y^2 - 2$ is not a function of x.

71. No, $y^2 - \frac{1}{4}x^2 = 4$ is not a function of x.

72. Yes, $y = |x + 4|$ is a function of x.

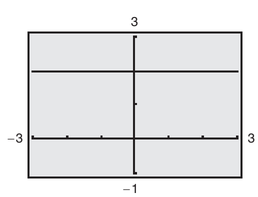
73.



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

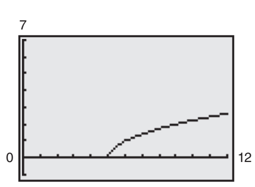
74.



Domain: $(-\infty, \infty)$

Range: $\{2\}$

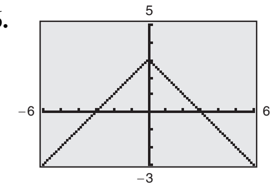
75.



Domain: $[-5, \infty)$

Range: $[0, \infty)$

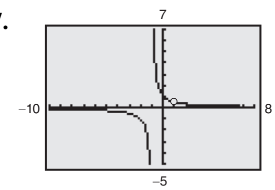
76.



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

77.

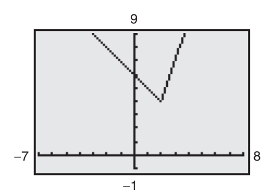


$$f(x) = \frac{x - 1}{x^2 - 1} = \frac{x - 1}{(x - 1)(x + 1)} = \frac{1}{x + 1},\ x \neq 1$$

Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range: $(-\infty, 0) \cup \left(0, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

78. $f(x) = \begin{cases} 6 - x, & x < 2 \\ 3x - 2, & x \geq 2 \end{cases}$



Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

79. $f(x) = 3x + 4$

(a) $f(1) = 3(1) + 4 = 7$

(b) $f(-5) = 3(-5) + 4 = -11$

(c) $f(x + 1) = 3(x + 1) + 4 = 3x + 7$

80. $f(x) = x^2 + 4x + 3$

(a) $f(0) = (0)^2 + 4(0) + 3 = 3$

(b) $f(3) = (3)^2 + 4(3) + 3 = 24$

(c) $f(x - 1) = (x - 1)^2 + 4(x - 1) + 3$
$= x^2 - 2x + 1 + 4x - 4 + 3$
$= x^2 + 2x$

81. (a) $f(x) + g(x) = (6 + x^2) + (3x - 5)$
$= x^2 + 3x + 1$

(b) $f(x) - g(x) = (6 + x^2) - (3x - 5)$
$= x^2 - 3x + 11$

(c) $f(x)g(x) = (6 + x^2)(3x - 5)$
$= 3x^3 - 5x^2 + 18x - 30$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{6 + x^2}{3x - 5}$

(e) $f(g(x)) = f(3x - 5)$
$= 6 + (3x - 5)^2$
$= 9x^2 - 30x + 31$

(f) $g(f(x)) = g(6 + x^2)$
$= 3(6 + x^2) - 5$
$= 3x^2 + 13$

82. (a) $f(x) + g(x) = 2x - 3 + \sqrt{x + 1}$

(b) $f(x) - g(x) = 2x - 3 - \sqrt{x + 1}$

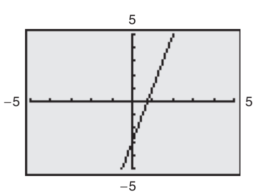
(c) $f(x)g(x) = (2x - 3)\sqrt{x + 1}$

(d) $\dfrac{f(x)}{g(x)} = \dfrac{2x - 3}{\sqrt{x + 1}}$

(e) $f(g(x)) = f(\sqrt{x + 1})$
$= 2\sqrt{x + 1} - 3$

(f) $g(f(x)) = g(2x - 3)$
$= \sqrt{(2x - 3) + 1}$
$= \sqrt{2x - 2}$

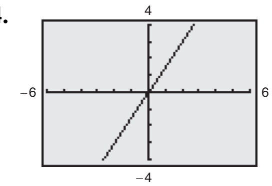
83.



$f(x)$ is one-to-one.

$f(x) = 4x - 3 = y$
$4y - 3 = x$
$4y = x + 3$
$y = \frac{1}{4}x + \frac{3}{4}$
$f^{-1}(x) = \frac{1}{4}x + \frac{3}{4}$

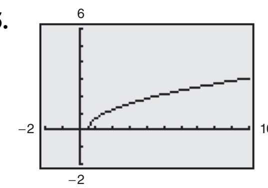
84.



$f(x)$ is one-to-one.

$f(x) = \frac{3}{2}x = y$
$\frac{3}{2}y = x$
$3y = 2x$
$y = \frac{2}{3}x$
$f^{-1}(x) = \frac{2}{3}x$

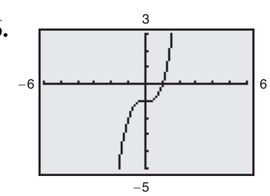
85.



$f(x)$ is one-to-one.

$f(x) = \sqrt{x - \frac{1}{2}}$
$y = \sqrt{x - \frac{1}{2}}$
$x = \sqrt{y - \frac{1}{2}}$
$x^2 = y - \frac{1}{2}$
$y = x^2 + \frac{1}{2}$
$f^{-1}(x) = x^2 + \frac{1}{2},\ x \ge 0$

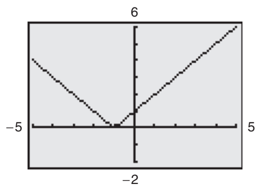
86.



$f(x)$ is one-to-one.

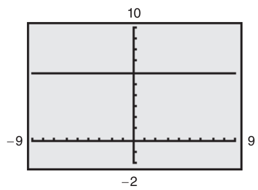
$f(x) = x^3 - 1 = y$
$y^3 - 1 = x$
$y^3 = x + 1$
$y = \sqrt[3]{x + 1}$
$f^{-1}(x) = \sqrt[3]{x + 1}$

87.



$f(x)$ does not have an inverse function.

88.



$f(x)$ does not have an inverse function.

89.

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$	0.6	0.96	0.996	?	1.004	1.04	1.4

$\lim_{x\to1}(4x-3)=1$

90.

x	2.9	2.99	2.999	3
$f(x)$	0.2564	0.2506	0.2501	?

x	3.001	3.01	3.1
$f(x)$	0.2499	0.2494	0.2439

$\lim_{x\to3}\dfrac{x-3}{x^2-2x-3}=0.25$

91.

x	−0.1	−0.01	−0.001	0
$f(x)$	0.2050	0.2042	0.2041	?

x	0.001	0.01	0.1
$f(x)$	0.2041	0.2040	0.2033

$\lim_{x\to0}\dfrac{\sqrt{x+6}-\sqrt{6}}{x}\approx0.204$

Note: $\lim_{x\to0}\dfrac{\sqrt{x+6}-\sqrt{6}}{x}=\dfrac{1}{2\sqrt{6}}$

92.

x	6.9	6.99	6.999	7
$f(x)$	−1.47	−14.33	−142.90	?

x	7.001	7.01	7.1
$f(x)$	142.82	14.24	1.39

$\lim_{x\to7}\dfrac{\dfrac{1}{x-7}-\dfrac{1}{7}}{x}$ does not exist.

93. $\lim_{x\to3}8=8$

94. $\lim_{x\to6}x^4=(6)^4=1296$

95. $\lim_{x\to2}(5x-3)=5(2)-3=7$

96. $\lim_{x\to5}(3x^2+4)=3(5)^2+4=79$

97. $\lim_{x\to-1}\dfrac{x+3}{6x+1}=\dfrac{-1+3}{6(-1)+1}=-\dfrac{2}{5}$

98. $\lim_{t\to-3}\dfrac{6t+5}{t+8}=\dfrac{6(-3)+5}{(-3)+8}=\dfrac{-13}{5}=-\dfrac{13}{5}$

99. $\lim_{t\to0^-}\dfrac{t^2+1}{t}=-\infty$

$\lim_{t\to0^+}\dfrac{t^2+1}{t}=\infty$

$\lim_{t\to0}\dfrac{t^2+1}{t}$ does not exist.

100. $\lim_{t\to2^-}\dfrac{t+1}{t-2}=-\infty$

$\lim_{t\to2^+}\dfrac{t+1}{t-2}=\infty$

$\lim_{t\to2}\dfrac{t+1}{t-2}$ does not exist.

101. $\lim_{x\to-2}\dfrac{x+2}{x^2-4}=\lim_{x\to-2}\dfrac{x+2}{(x+2)(x-2)}$

$=\lim_{x\to-2}\dfrac{1}{x-2}$

$=-\dfrac{1}{4}$

102. $\lim_{x\to4}\dfrac{x^2-6x-8}{x-4}=\lim_{x\to4}\dfrac{(x-4)(x+2)}{x-4}$

$=\lim_{x\to4}(x+2)$

$=6$

103. $\lim_{x\to 0^+}\left(x - \frac{1}{x}\right) = \lim_{x\to 0^+}\frac{x^2 - 1}{x} = -\infty$

104. $\lim_{x\to 1/2^-}\frac{-x}{6x - 3} = \frac{\frac{1}{2}}{6\left(\frac{1}{2}\right) - 3} = \frac{\frac{1}{2}}{3 - 3} = \frac{\frac{1}{2}}{0} = \infty$

105.
$$\lim_{x\to 0}\frac{\sqrt{3} - \sqrt{x + 3}}{x} = \lim_{x\to 0}\frac{\sqrt{3} - \sqrt{x + 3}}{x}\cdot\frac{\sqrt{3} + \sqrt{x + 3}}{\sqrt{3} + \sqrt{x + 3}}$$
$$= \lim_{x\to 0}\frac{3 - (x + 3)}{x\left(\sqrt{3} + \sqrt{x + 3}\right)}$$
$$= \lim_{x\to 0}\frac{-x}{x\left(\sqrt{3} + \sqrt{x + 3}\right)}$$
$$= \lim_{x\to 0}\frac{-1}{\sqrt{3} + \sqrt{x + 3}}$$
$$= -\frac{1}{2\sqrt{3}} = -\frac{\sqrt{3}}{6}$$

106.
$$\lim_{s\to 0}\frac{\left(1/\sqrt{1 + s}\right) - 1}{s} = \lim_{s\to 0}\frac{1 - \sqrt{1 + s}}{s\sqrt{1 + s}}\cdot\frac{1 + \sqrt{1 + s}}{1 + \sqrt{1 + s}}$$
$$= \lim_{s\to 0}\frac{1 - (1 + s)}{s\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)}$$
$$= \lim_{s\to 0}\frac{-1}{\sqrt{1 + s}\left(1 + \sqrt{1 + s}\right)}$$
$$= -\frac{1}{2}$$

107. $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} x + 5, & x \neq 0 \\ 3, & x = 0 \end{cases}$

$\lim_{x\to 0} f(x) = \lim_{x\to 0}(x + 5) = 0 + 5 = 5$

108. $\lim_{x\to -2} f(x)$, where $f(x) = \begin{cases} \frac{1}{2}x + 5, & x < -2 \\ -x + 2, & x \geq -2 \end{cases}$

$\lim_{x\to -2} f(x) = -x + 2 = -(-2) + 4 = 4$

109.
$$\lim_{\Delta x\to 0}\frac{(x + \Delta x)^3 - (x + \Delta x) - (x^3 - x)}{\Delta x} = \lim_{\Delta x\to 0}\frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x - \Delta x - x^3 + x}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - \Delta x}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\left[3x^2 + 3x\Delta x + (\Delta x)^2 - 1\right]$$
$$= 3x^2 - 1$$

110.
$$\lim_{\Delta x\to 0}\frac{\left[1 - (x + \Delta x)^2\right] - (1 - x^2)}{\Delta x} = \lim_{\Delta x\to 0}\frac{1 - x^2 - 2x\Delta x - (\Delta x)^2 - 1 + x^2}{\Delta x}$$
$$= \lim_{\Delta x\to 0}\frac{\Delta x(-2x - \Delta x)}{\Delta x}$$
$$= \lim_{\Delta x\to 0}(-2x - \Delta x) = -2x$$

111. $f(x) = x + b$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

112. $f(x) = x^2 + 3x + 2$ is continuous on $(-\infty, \infty)$ because the domain of f consists of all real x.

113. $f(x) = \frac{1}{(x + 4)^2}$ is continuous on the intervals $(-\infty, -4)$ and $(-4, \infty)$ because the domain of f consists of all real numbers except $x = -4$. There is a discontinuity at $x = -4$ because $f(4)$ is not defined.

114. $f(x) = \dfrac{x+2}{x}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$ because the domain of f consists of all real numbers except $x = 0$. There is a discontinuity at $x = 0$ because $f(0)$ is not defined.

115. $f(x) = \dfrac{3}{x+1}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real numbers except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

116. $f(x) = \dfrac{x+1}{2x+2}$ is continuous on the intervals $(-\infty, -1)$ and $(-1, \infty)$ because the domain of f consists of all real number except $x = -1$. There is a discontinuity at $x = -1$ because $f(-1)$ is not defined.

117. $f(x) = \sqrt{x-8}$ is continuous on the interval $[8, \infty)$ because the domain of f consists of all real numbers $x \ge 8$. For all values of $c > 8$, $F(c)$ is defined, the limit exists as $x \to c$, and $f(c) = \lim\limits_{x \to c} f(x)$.

$\left[\text{Note: } f \text{ is continuous at } x = 8 \text{ since } f(8) = \lim\limits_{x \to 8^+} f(x).\right]$

118. $f(x) = \sqrt{5-x}$ is continuous on the interval $(-\infty, 5]$ because the domain of f consists of all real numbers $x \le 5$. For all values of $c < 5$, $F(c)$ is defined, the limit exists as $x \to c$, and $f(c) = \lim\limits_{x \to c} f(x)$.

$\left[\left[\text{Note: } f \text{ is continuous at } x = 5 \text{ since } \lim\limits_{x \to 5^-} f(x).\right]\right]$

119. $f(x) = [\![x + 3]\!]$ is continuous on all intervals of the form $(c, c+1)$, where c is an integer. There are discontinuities at all integer values c because $\lim\limits_{x \to c} f(x)$ does not exist.

120. $f(x) = [\![x]\!] - 2$ is continuous on all intervals of the form $(c, c+1,)$ where c is an integer. There are discontinuities at all integer values c because $\lim\limits_{x \to c} f(x)$ does not exist.

121. $f(x) = \begin{cases} x, & x \le 0 \\ x+1, & x > 0 \end{cases}$ is continuous on the intervals $(-\infty, 0)$ and $(0, \infty)$. There is a discontinuity at $x = 0$ because $\lim\limits_{x \to 0} f(x)$ does not exist.

122. $f(x) = \begin{cases} x, & x \le 0 \\ x^2, & x > 0 \end{cases}$ is continuous on $(-\infty, \infty)$ because $f(0)$ is defined, $\lim\limits_{x \to 0} f(x)$ exists, and $\lim\limits_{x \to 0} f(x) = f(0)$.

123. $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} (-x + 1) = -2$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (ax - 8) = 3a - 8$

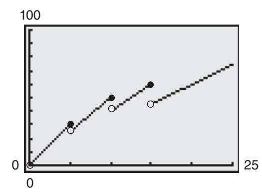
So, $-2 = 3a - 8$ and $a = 2$.

124. $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} (x + 1) = 2$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} (2x + a) = 2 + a$

So, $2 = 2 + a$ and $a = 0$.

125. (a)



Explanations will vary. The function is defined for all values of x greater than zero. The function is not continuous at $x = 5$, $x = 10$, and $x = 15$.

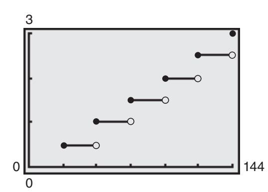
(b) $C(10) = 4.99(10) = \$49.90$

126. $\lim\limits_{t \to 2^-} S(t) = 41{,}400$

$\lim\limits_{t \to 2^+} S(t) = 42{,}849$

The limit of S as t approaches 2 does not exist.

127. (a)

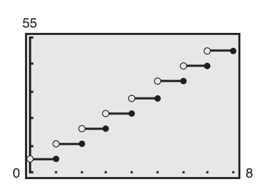


The function is not continuous at $x = 24n$, where n is a positive integer.

(b) When $x = 1500$, $A = \$31$.

128. (a) $C = 32.3 - 2.9[\![1 - x]\!]$

(b)



Chapter 1 Test Yourself

1. (a) $d = \sqrt{(-4-1)^2 + (4-(-1))^2}$

$= \sqrt{(-5)^2 + (5)^2} = \sqrt{50} = 5\sqrt{2}$

(b) Midpoint $= \left(\frac{1+(-4)}{2}, \frac{-1+4}{2}\right) = \left(-\frac{3}{2}, \frac{3}{2}\right)$

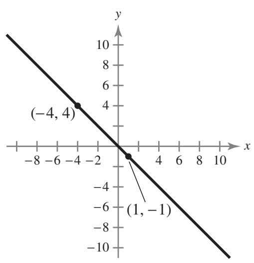
(c) $m = \frac{4-(-1)}{-4-1} = \frac{5}{-5} = -1$

(d) $y - (-1) = -1(x - 1)$

$y + 1 = -x + 1$

$y = -x$

(e)



2. (a) $d = \sqrt{\left(0 - \frac{5}{2}\right)^2 + (2-2)^2}$

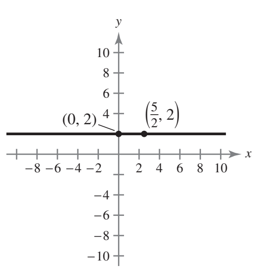
$= \sqrt{\left(-\frac{5}{2}\right)^2 + 0^2} = \frac{5}{2}$

(b) Midpoint $= \left(\frac{\frac{5}{2}+0}{2}, \frac{2+2}{2}\right) = \left(\frac{5}{4}, 2\right)$

(c) $m = \frac{2-2}{0-\frac{5}{2}} = 0$

(d) Horizontal line: $y = 2$

(e)



3. (a) $d = \sqrt{(-4-2)^2 + (1-3)^2}$

$= \sqrt{(-6)^2 + (-2)^2} = \sqrt{40}$

$= 2\sqrt{10}$

(b) Midpoint $= \left(\frac{2+(-4)}{2}, \frac{3+1}{2}\right)$

$= \left(-\frac{2}{2}, \frac{4}{2}\right)$

$= (-1, 2)$

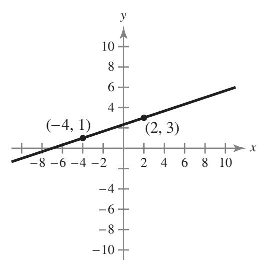
(c) $m = \frac{1-3}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$

(d) $y - 3 = \frac{1}{3}(x - 2)$

$y - 3 = \frac{1}{3}x - \frac{2}{3}$

$y = \frac{1}{3}x + \frac{7}{3}$

(e)



4. $65 - 2.1x = 43 + 1.9x$

$-4x = -22$

$x = 5.5$

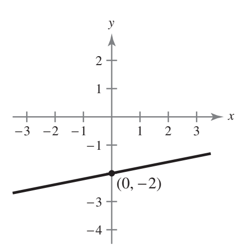
Equilibrium point $(x, p) = (5.5, 5500)$

The equilibrium point occurs when the demand and supply are each 5500 units.

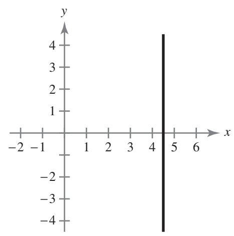
5. $m = \frac{1}{5}$

When $x = 0$: $y = \frac{1}{5}(0) - 2 = -2$

y-intercept: $(0, -2)$



6. The line $x - \frac{9}{2} = 0 \Rightarrow x = \frac{9}{2}$ is vertical, so its slope is undefined, and it has no y-intercept.

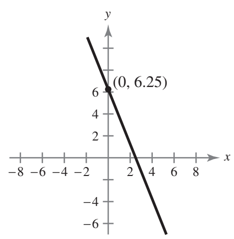


7. $y = -2.5x + 6.25$

$m = -2.5$

When $x = 0$: $y = -2.5(0) + 6.25 = 6.25$

y-intercept: $(0, 6.25)$



8. The slope of the given line $-6x + y = 3 \Rightarrow y = 6x + 3$ is $m = 4$. The slope of the perpendicular line is $m = -\frac{1}{6}$.

Using the point $(-3, -1)$ and $m = -\frac{1}{4}$, the equation is:

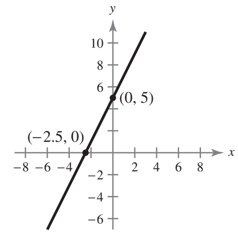
$$y - (-1) = -\frac{1}{6}(x - (-3))$$
$$y + 1 = -\frac{1}{6}(x + 3)$$
$$y + 1 = -\frac{1}{6}x - \frac{1}{2}$$
$$y = -\frac{1}{6}x - \frac{3}{2}.$$

9. The slope of the given line $5x - 2y = 8 \Rightarrow -2y = -5x + 8 \Rightarrow y = \frac{5}{2}x - 4$ is $m = \frac{5}{2}$. The slope of the line parallel is $m = \frac{5}{2}$.

Using the point $(2, 1)$ and $m = \frac{5}{2}$, the equation is:

$$y - 1 = \frac{5}{2}(x - 2)$$
$$y - 1 = \frac{5}{2}x - 5$$
$$y = \frac{5}{2}x - 4$$

10. (a)



(b) Domain: $(-\infty, \infty)$

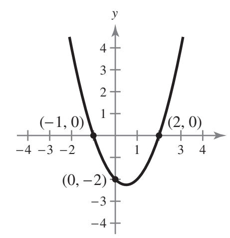
Range: $(-\infty, \infty)$

(c)

x	-3	-2	3
$f(x)$	-1	1	11

(d) The function is one-to-one.

11. (a)



(b) Domain: $(-\infty, \infty)$

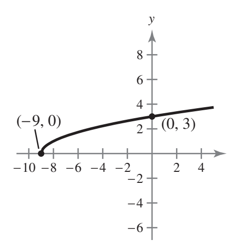
Range: $\left[-\frac{9}{4}, \infty\right)$

(c)

x	-3	-2	3
$f(x)$	10	4	4

(d) The function is not one-to-one.

12. (a)



(b) Domain: $[-9, \infty)$

Range: $[0, \infty)$

(c)

x	-3	-2	3
$f(x)$	$\sqrt{6}$	$\sqrt{7}$	$\sqrt{12} = 2\sqrt{3}$

(d) The function is one-to-one.

13. $f(x) = 4x + 6 = y$

$$4y + 6 = x$$
$$4y = x - 6$$
$$y = \frac{1}{4}x - \frac{3}{2}$$
$$f^{-1}(x) = \frac{1}{4}x - \frac{3}{2}$$

14. $f(x) = \sqrt[3]{8 - 3x} = y$

$\sqrt[3]{8 - 3y} = x$

$8 - 3y = x^3$

$-3y = x^3 - 8$

$y = -\frac{1}{3}x^3 + \frac{8}{3}$

$f^{-1}(x) = -\frac{1}{3}x^3 + \frac{8}{3}$

15. $\lim_{x \to 0} \dfrac{x - 2}{x + 2} = \dfrac{0 - 2}{0 + 2} = -1$

16.

x	4.9	4.99	4.999	5.001	5.01	5.1
$f(x)$	–99	–999	–9999	10,001	1001	101

$\lim_{x \to 5^-} \dfrac{x + 5}{x - 5} = -\infty$

$\lim_{x \to 5^+} \dfrac{x + 5}{x - 5} = \infty$

$\lim_{x \to 5} f(x)$ does not exist.

17. $\lim_{x \to -3} \dfrac{x^2 + 2x - 3}{x^2 + 4x + 3} = \lim_{x \to -3} \dfrac{(x - 1)(x + 3)}{(x + 1)(x + 3)}$

$= \lim_{x \to -3} \dfrac{x - 1}{x + 1}$

$= 2$

18.

x	–0.01	–0.001	–0.0001	0.0001	0.001	0.01
$f(x)$	0.16671	0.16667	0.16667	0.16667	0.16666	0.16662

$\lim_{x \to 0} \dfrac{\sqrt{x + 9} - 3}{x} \approx 0.16667$

19. $f(x) = \dfrac{x^2 - 36}{x - 6}$ is continuous on the intervals $(-\infty, 6)$ and $(6, \infty)$ because the domain of f consists of all real numbers except $x = 6$. There is a discontinuity at $x = 6$ because $f(6)$ is not defined.

20. $f(x) = \sqrt{5 - x}$ is continuous on the interval $(5, \infty)$ because the domain of f consists of all $x > 5$.

21. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (1 - x) = 0$

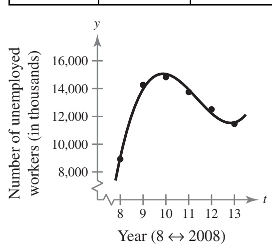
$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x - x^2) = 0$

So, $\lim_{x \to 1} f(x) = 0$.

Because $f(1)$ is defined, $\lim_{x \to 1} f(x)$ exists, and $\lim_{x \to 1} f(x) = f(1)$, the function is continuous on the interval $(-\infty, \infty)$.

22. (a)

Year	2008	2009	2010	2011	2012	2013
Actual	8924	14,265	14,825	13,747	12,506	11,460
Model	9026	13,956	15,046	13,924	12,219	11,558



The model fits the data well.

(b) Let $t = 18$.

$y = 271.343(18)^3 - 9246.20(18)^2 + 103{,}234.1(18) - 364{,}018$

$\approx 80{,}899.376$ thousand

In 2018, the number of unemployed workers will by 80,899,376. This prediction is invalid because this would represent an increase of over 600% in a five-year period, which is unreasonable.